Advanced Pipeline Design

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ABSTRACT

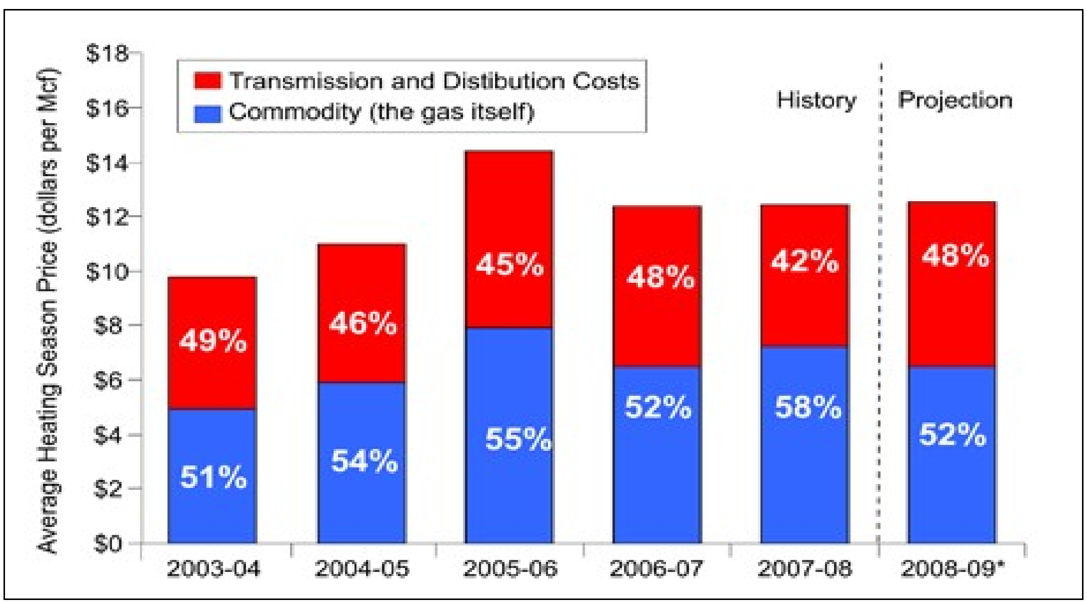
The material presented in this report is an investigation of conventional natural gas pipeline optimization. First, a comprehensive study on various natural gas hydraulic equations was conducted. The results of this study proved that current hydraulic equations produce large amounts of error when modeling a pipeline. This error is unacceptable due to the high costs associated with this hot commodity. Furthermore, the conventional economical analysis using J-curves has been proven to be extremely time consuming if accurate results are desired. Thus, the implementation of a new methodology of optimization in natural gas pipelines is extremely necessary. This report expands upon a state-of-the-art discrete mathematical optimizer and applies the tool to a ramified natural gas pipeline case study.

Introduction

In these times of volatile natural gas prices, it is imperative to minimize all costs associated with this hot commodity. The demand for natural gas has increased each year due to the increase of world population and the industrialization of nations such as China and India. This past year, the United States alone consumed approximately 1.5 to 2.5 million cubic feet (MMscf) of natural gas every month. Additionally, approximately 97% of this consumption comes from gas piped straight from the well-head source[1]. These circumstances have caused the increasing natural gas price trend that the world has experienced in the last few decades.

The price of natural gas that the consumer pays is made up of two components: the price of the actual commodity and the cost of transmitting and distributing the gas. Shockingly, transmission and distribution is estimated to make up 48% of the consumer's price of gas as shown in the following diagram (EIA):

Figure 1: Cost of Natural Gas

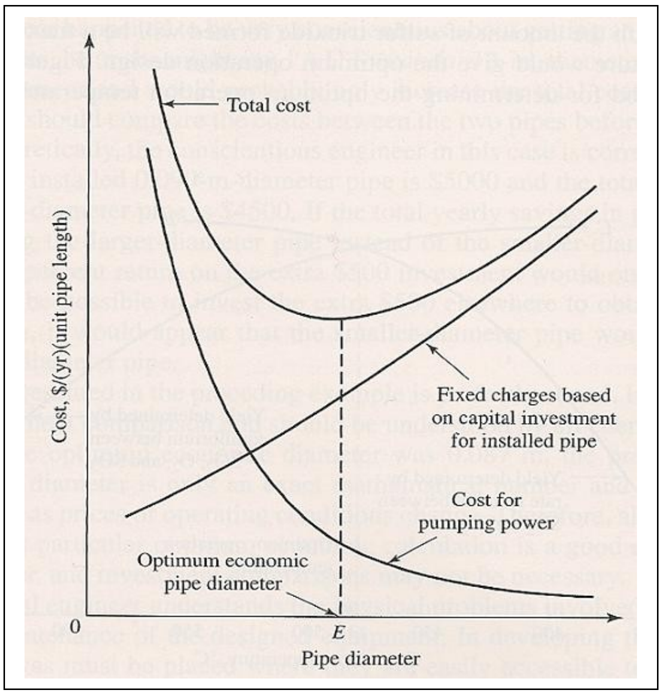


That means that nearly half of the cost of natural gas comes from transporting the commodity from the wellhead, to the consumer's meter. Also, natural gas has a potential increase in demand due to it being a clean source of fuel for electricity plants and compressed natural gas automobiles

(Pickens). These facts emphasize the importance of designing and operating natural gas pipelines at their economically optimal levels.

The primary costs associated with the transmission and distribution of natural gas are the initial capital costs and the costs of operating the pipeline. In theory, a perfectly designed pipeline would minimize the combination of the annual costs of compression and the initial fixed charges for the pipeline. The following diagram demonstrates exactly how this could be accomplished for a extremely simple case (Peters):

Figure 2: Basic Pipeline Optimization



This figure shows that the additional cost of operating the compressor combined with the fixed charges associated with the pipeline will create the function representing the total cost of the pipeline. In a simple example, one could easily add the two functions together; then take the derivative of the new combined function with respect to diameter. Then set the resultant equal to zero and solve for the diameter. While that sounds easy for simple systems, it becomes exponentially impossible to recreate for complex systems. In real, complex pipe networks, the operating costs are made up of many complex functions. Likewise, cost functions for the fixed capital investment are very complex and also made up of several factors. Thus, the resultant total cost function may not be a simple curve, and it could be a polynomial function with many local minimums and one global optimal minimum total cost. One could understand why this problem could be impossible to complete by hand without using several risky assumptions.

Furthermore, the initial capital costs of a pipeline are dominated by the costs of the pipe and compressor stations. In fact, the pipe and compressors make up 96% of the construction materials (Menon). Thus, it is essential for pipeline designers to build the pipes and compressors with precise economic specifications. Likewise, a major cost of operating a pipeline is the cost of fuel consumed by compressors that are pushing the gas down the pipeline. The work associated with operating a compressor and the initial cost of pipe are interrelated by the diameter of the pipe. Therefore, designing a pipe with the appropriate diameter is crucial to the optimization of the pipeline.

Natural Gas Hydraulics

In order to minimize the annual compression costs associated with a pipeline, the designer must accurately estimate the pressure drop that will occur throughout the pipe. Conventionally, the pressure drop has been calculated using numerous correlations derived from the mechanical energy balance (Bernoulli Equation) that relates pressure drop to flow rate:

$$u\,du + \frac{dP}{\rho} + g\,dH + f\frac{dx}{D}\frac{u^2}{2} = 0 \tag{1}$$

where u is the velocity of the fluid, P is the pressure, ρ is the density of the fluid, g is the gravity constant, dH is the height variation, f is the Darcy friction factor, dx is the change in length of the pipe, and D is the diameter. The first term in Equation 1 is the kinetic energy (KE) term, the second term represents the pressure drop of the system, the third term is the potential energy (PE) of the pipe, and the final term is the head loss or friction loss due to the inside walls of the pipe. Since natural gas is not an incompressible, Newtonian fluid, the assumption of constant density cannot be applied to the energy balance. Therefore, the integration becomes quite difficult. Thus, in order to make the integration less complicated, average values of compressibility, temperature, and pressure are utilized.

Compared to the frictional forces, the kinetic energy brought to the system by the movement of the gas molecules is considered negligible and therefore the integration of the KE goes to zero. Next, the pressure drop term can be integrated between two points of pipe as follows:

$$\int_{P_1}^{P_2} \rho \, dP = \int_{P_1}^{P_2} \frac{PM}{z\,R\,T} dP = \frac{M}{z_{avg} R T_{avg}} \int_{P_1}^{P_2} P dP = \frac{M}{z_{avg} R T_{avg}} \frac{(P_2^2 - P_1^2)}{2} \tag{2}$$

where z is the compressibility of the gas, R is the gas constant, and M is the molecular weight of the gas. The aforementioned PE term is likewise integrated:

$$\int_{z_1}^{z_2} \rho^2 g \, dz = \int_{z_1}^{z_2} \left(\frac{PM}{z\,R\,T}\right)^2 g \, dz = \frac{g P_{avg}^2 M^2}{z_{avg}^2 R^2 T_{avg}^2} (z_2 - z_1) \tag{3}$$

Finally, the friction loss term is integrated between points 1 and 2:

$$\int_{x_1}^{x_2} \frac{f\,C^2}{2\,D} dx = f \frac{(x_2 - x_1)}{D} \frac{C^2}{2} = f \frac{L}{D} \frac{C^2}{2} \tag{4}$$

where

$$C = \rho_{st} v = \frac{m_{st}}{A} \tag{5}$$

where L is the length of the pipe, ρ_{st} is the density of the gas at standard conditions, m_{st} is the mass flowrate of the gas at standard conditions, and A is the cross-sectional area of the pipe.

Since C is defined as above, the C^2 can be defined as:

$$C^2 = \frac{\rho_{st}^2 \, Q_{st}^2}{A^2} = \frac{16 P_{st}^2 M^2 Q_{st}^2}{\pi^2 D^4 z_{st}^2 R^2 T_{st}^2} \tag{6}$$

where Q is the volumetric flowrate of the gas and M is defined as:

$$M = d \, M_{air} \tag{7}$$

with d being defined as the gas density relative to air and M_{air} is the molecular weight of air equal to 28.9625 ≈ 29.

The combination of the integrated terms above and solving the equation for flowrate produces the following equation known as the General Flow Equation (Coelho):

$$\dot{Q}_{st} = \frac{\pi}{4\ \rho_{st}} \left(\frac{29\ d}{z_{avg} \bar{R}\, T_{avg} L} \left[(P_1^2 - P_2^2) - 2\ P_{avg} \rho_{avg} g\ (H_2 - H_1) \right] \right)^{1/2} \frac{D^{2.5}}{\sqrt{f}} \qquad (8)$$

This equation, when combined with different definitions of the value of the friction factor, produces the conventional equations studied in this report. An in depth investigation of the following equations, which are all derived from the General Flow Equation, was conducted.

1. Colebrook-White
2. Modified Colebrook-White
3. AGA
4. Panhandle
5. Weymouth
6. IGT
7. Spitzglass
8. Mueller
9. Fritzsche
10. Etc.

According to the results of an in depth analysis attached in Appendix A; the industrial equations can produce large amounts of error when compared to a process simulator (Pro/II) analysis with the same pipe specifications and conditions. The following table shows the results of the equation study:

Table 1: Conventional Hydraulic Equation Analysis

Equation Name	Range of Error
Panhandle	3.5 – 10%
Colebrook	2.4 – 10%
Modified-Colebrook	1.0 – 8.8%
AGA	0.2 – 15%
Weymouth	39 – 59%
IGT	7.6 – 17%
Spitzglass	88 – 147%
Mueller	13 – 20%
Fritzsche	40 – 52%

The error produced by these equations is due to the assumptions used to derive the equation from the General Flow Equation. Also, the extremely large amounts of error occurred when the equations were applied outside their intended pipeline environment. This error could directly affect theoretical optimal pipeline diameter and cause it to be significantly different from the actual optimal pipe diameter.

In order to estimate the cost of error per year, the following rates and assumptions were implemented. The 2008 average price of natural gas at the wellhead was approximately $8 per Mscf (EIA). At this rate and incorporating a natural gas pipeline flowing 200 MMscfd, and considering that 4% of the flowrate is used for compressor fuel. Thus, a 1% error correlates to just over $220 worth of natural gas wasted in fuel costs per year per compressor station. Another assumption is that the pipe and compressor are running 350 days a year. This emphasizes the importance of accuracy when estimating the optimal specifications and working conditions of a pipeline. Thus, it is economically unacceptable to implement the current industry standard pressure drop and flowrate correlations that produce 5% error at best.

Mathematical Model General Hydraulic Equation

The previous section proved that the conventional equations used to calculate the pressure drop as a function of the gas flowrate are inadequate. Therefore, a more generic form of the equation was used in a mathematical model in order to eliminate the error produced by faulty friction assumptions. To obtain a generic equation, a two parameter equation was created from the General Flow Equation:

$$\dot{Q}_{st} = \frac{\pi}{4\ \rho_{st}} \left(\frac{29\ d}{z_{avg} \bar{R}\, T_{avg} L} \left[(P_1^2 - P_2^2) - 2\ P_{avg} \rho_{avg} g\ (H_2 - H_1) \right] \right)^{1/2} \frac{D^{2.5}}{\sqrt{f}} \tag{9}$$

where,

$$C' = \frac{\pi}{4\rho_{st}} \sqrt{\frac{29}{R}} \tag{10}$$

Squaring both sides of the equation above and rearranging:

$$\frac{Q^2 L}{D^5} = \frac{C'^2 (P_1^2 - P_2^2) - 2 * P_{avg} \rho_{avg} g (H_2 - H_1)}{d T_{avg} z_{avg} f} \tag{11}$$

In order to simplify this equation, two parameters are created (A and B) that equal the following:

$$A = \frac{C'^2}{d * T_{avg} z_{avg} f} \tag{12}$$

and

$$B = \frac{2 P_{avg} \rho_{avg}}{d T_{avg} z_{avg} f} \tag{13}$$

Finally, the two parameter generic equation that is used in optimization modeling system results in:

$$\frac{Q^2 L}{D^5} = A(P_1^2 - P_2^2) - B \Delta H \tag{14}$$

This generic form of the flowrate and pressure drop correlation is utilized in the General Algebraic Modeling System (GAMS) where it iterates the parameters up to 1,000,000 times and is an interface for solvers to converge into accurate results. The mathematical program uses this function to determine the diameter of a pipeline, and when combined with the economic functions of the system, it will converge the model to economically optimal diameters and operating conditions.

Economic Analysis

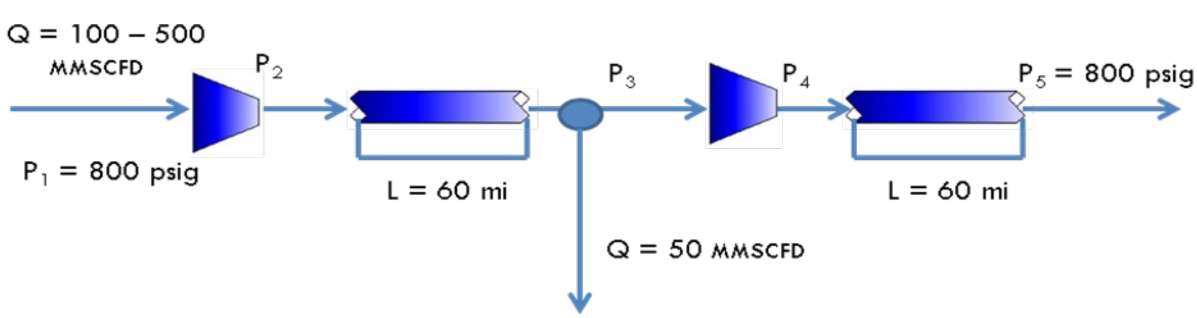
Conventional Analysis

The overall goal of pipeline optimization is to minimize the net present total annualized cost (NPTAC) associated with a pipeline. A conventional method of accomplishing that task is to perform J-Curve analysis on the pipeline network. However, this type of analysis has been proven to be inaccurate and very time consuming. Furthermore, Appendix B has a summary of J-Curve analysis. Therefore, an improved method of optimization of pipelines is something that would be very useful for the natural gas industry.

J-Curve Analysis Compared to Mathematical Programming Analysis

In order to justify continuing the research of the mathematical programming analysis of pipeline optimization; a direct comparison of a pipeline optimized by the mathematical program is made to the pipeline optimized using the J-Curve analysis. This is necessary in order to determine which method provided the most accurate results and which was the least time consuming. Therefore, the following simple pipeline system was economically optimized using both methods of analysis:

Figure 3: Simple Two Segment Pipeline



The system is set up with two pipe segments, two compressors, and a consumer point in between. The supplier volume varied between 100 to 500 MMscfd. The consumer that separates the two compression points consumes only 50 MMscfd of the gas, while the rest travels through the second compressor and pipe segment. Next, P_1 and P_5 are set at 800 psig, while the pressure at P_3, the midway consumer, would be optimally chosen between 750, 800, and 850 psig. Only three pressures were evaluated in order to keep the amount of simulations run for the J-Curve analysis to a minimum. This set up nicely for the discrete linear model which analyzes a set of pressure instead of pinpointing the actual optimum pressures in the pipeline.

In this example, both the pipe segments have distinct optimums. So, the first problem that arose while performing the J-Curve method was the meticulous process of picking which segment to analyze first, and then optimizing the other segment. In order to be sure that those segments were the actual optimum, the other segment had to be optimized first and then each optimization was compared to each other.

J-Curve Analysis on Two-Segment Pipeline

The J-Curve method of optimization analysis produced the following results for a flowrate of 300 MMscfd:

Figure 4: Segment 1 Analyzed at 850 psig

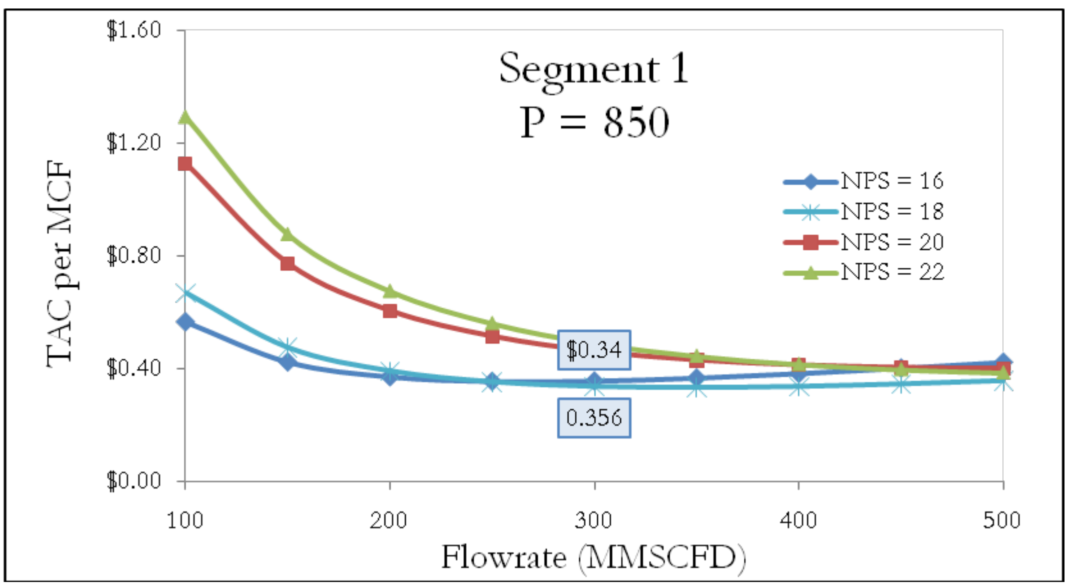


Figure 5: Segment 1 Analyzed at 800 psig

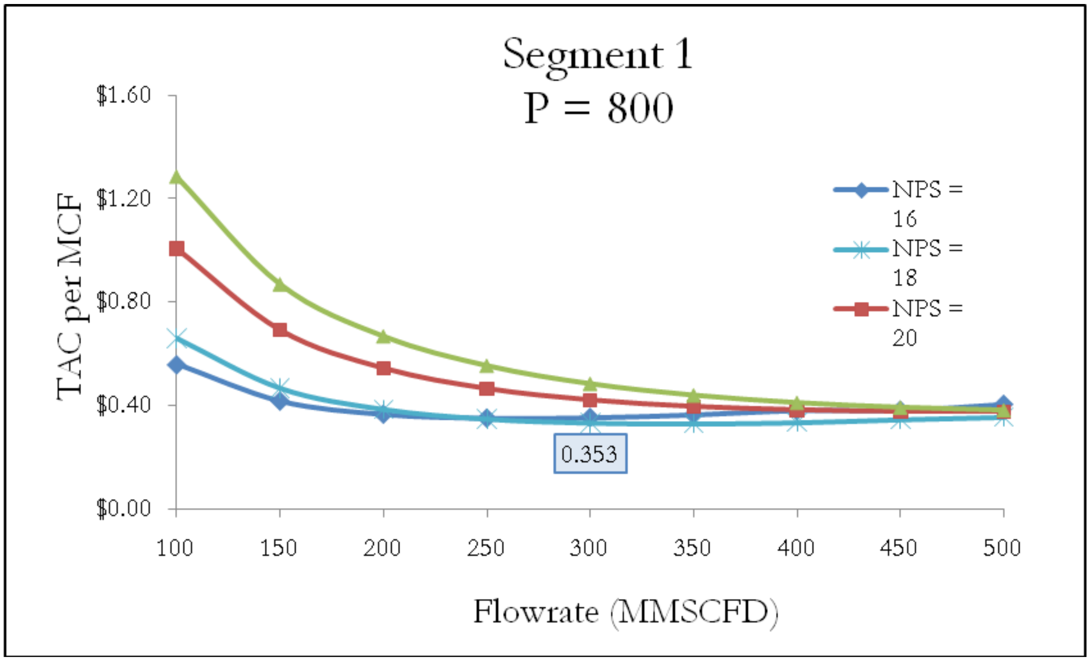
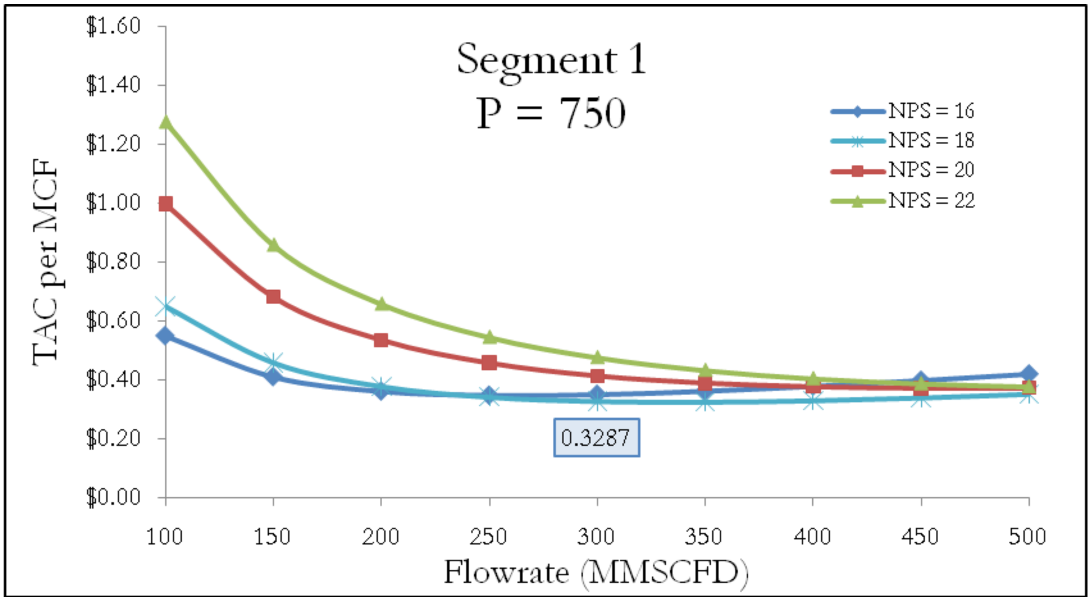


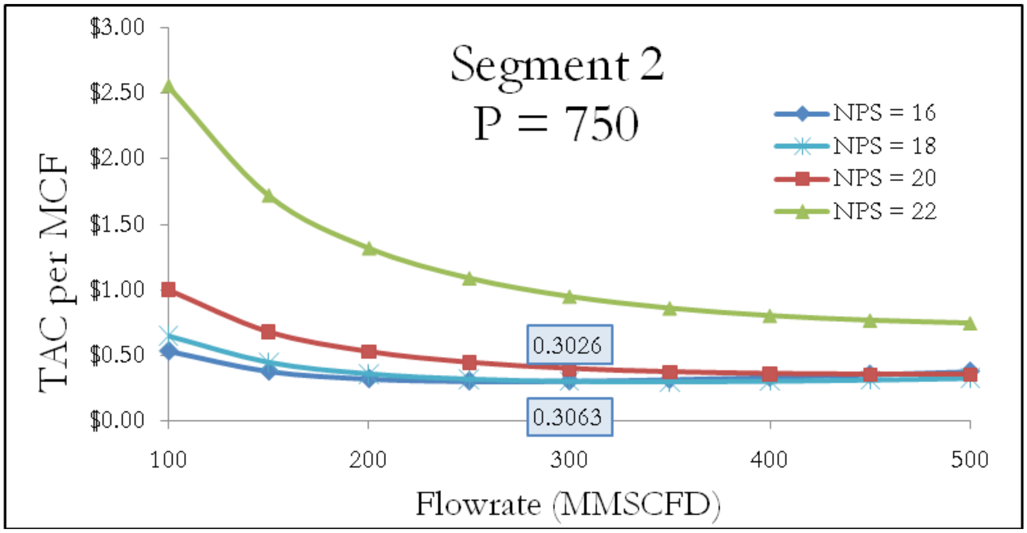
Figure 6: Segment 1 Analyzed at 750 psig



Figures 4, 5, and 6 show that, at all three pressures, the 18" NPS produced the smallest TAC per Mcf. Also, the results suggest that the 750 psig should be the suction pressure at the consumer point P_3. The optimal point for segment one produces a TAC of 0.3287\$/Mcf.

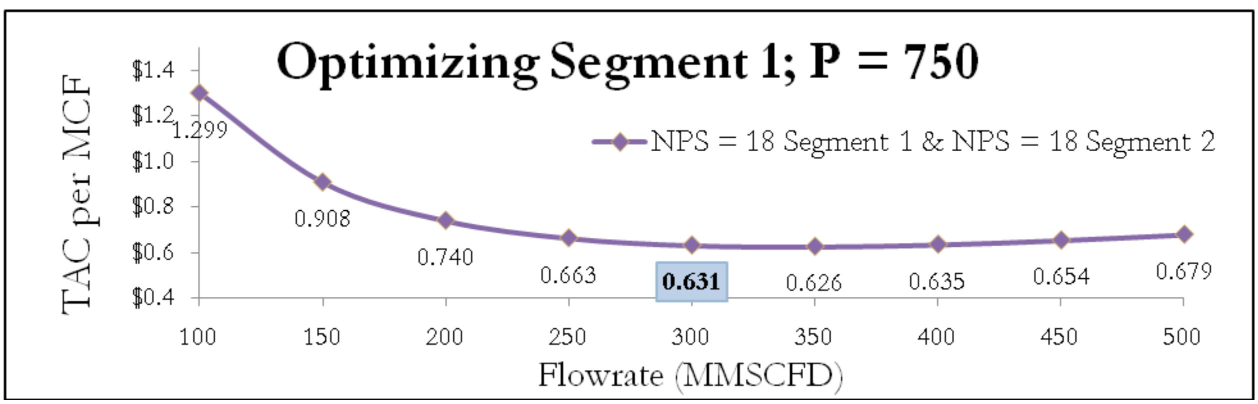
Since the optimal pressure at P_3, starting with segment 1, was found to be 750 psig; it now must be applied to the second segment and optimized in a similar method.

Figure 7: Segment 2 Analyzed at 750 psig



Now, the two segments must be combined in order to determine the overall TAC for the entire system. This is shown below in Figure 8:

Figure 8: Combined Optimization with Starting with Segment 1



This entire process must now be repeated and the optimization must start with segment 2 in order to indicate the actual optimal TAC. The results for the optimization using J-Curves and starting with the second pipe segment are shown below in Figure 9:

Figure 9: Combined Optimization with Starting with Segment 2

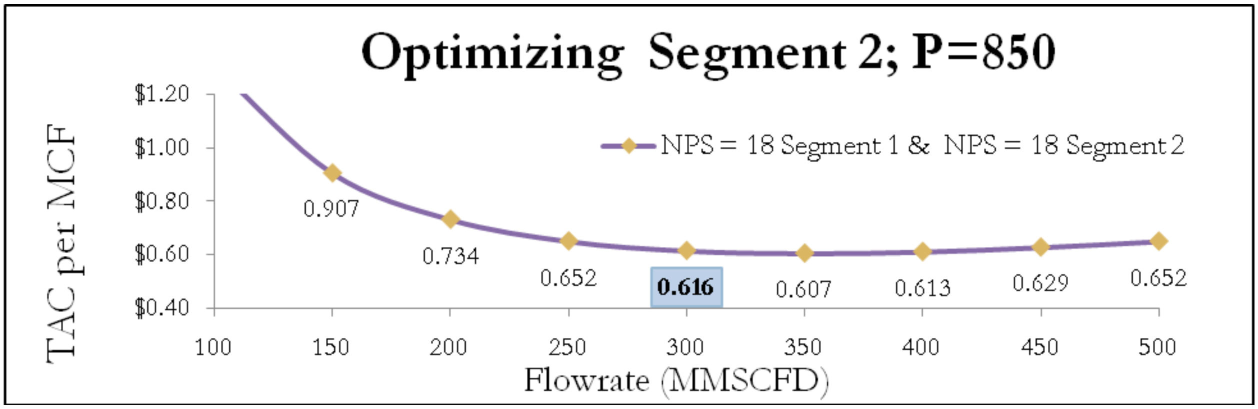
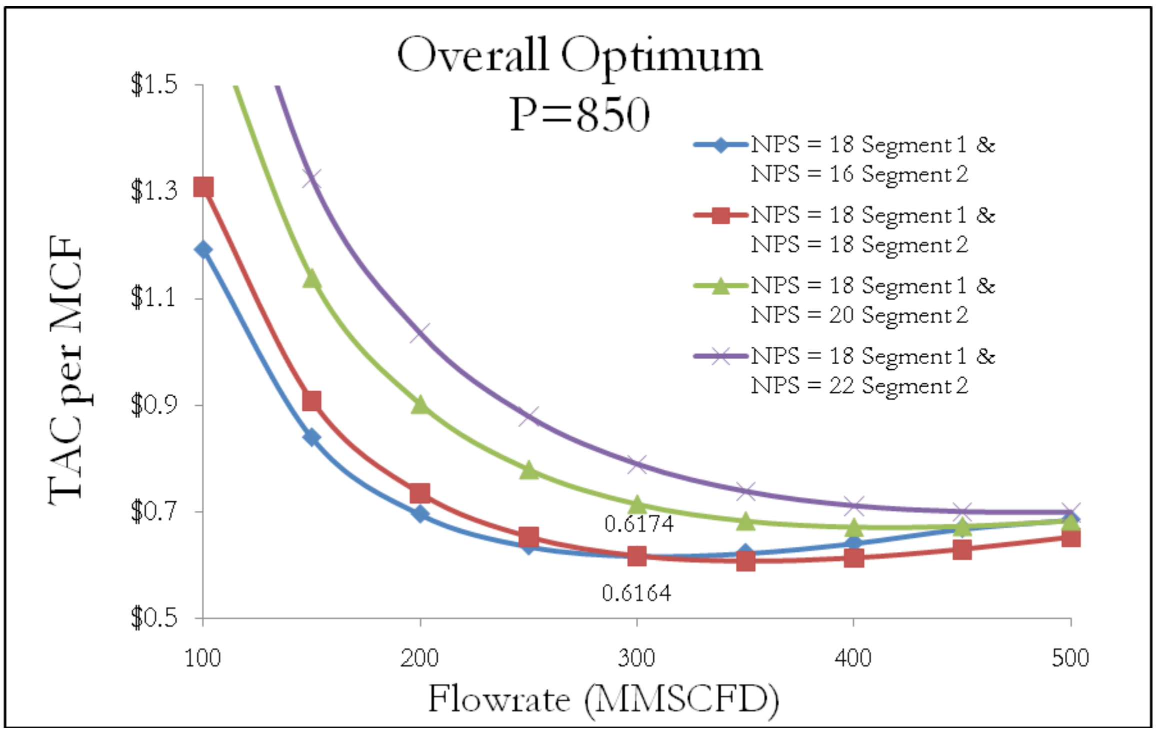


Figure 9 indicates that P_3 should actually be 850 psig due to the fact that it produced a smaller TAC per Mcf than P_3 at 750 psig which was the value found by starting the optimization with the first pipe segment. Thus, the overall optimum is shown in the following figure:

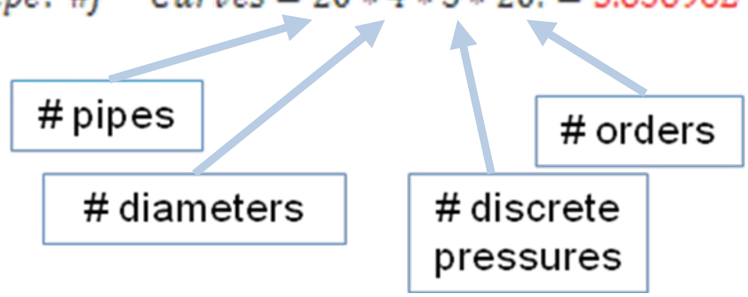
Figure 10: Overall Optimum for 2 Segment Pipeline



Therefore, with the complete J-Curve analysis for this simple system, which involved constructing 48 curves and running the process simulator 432 times, produced the optimal conditions of P_3 equaling 850 psig and both segments 1 and 2 to have 18 NPS pipe. Pro/II process simulations were used to construct the curves instead of the conventional pipeline hydraulic equations in order to eliminate any error that was discovered in Appendix A.

It is shown below that this required the construction of 48 J-Curves. It is also seen that the number of J-Curves required increases exponentially as more pipes are added to the system.

$$1\,pipe\!:\ \#J - Curves = 1 * 4 * 3 * 1! = 12$$
$$\boxed{2\,pipe\!:\ \#J - Curves = 2 * 4 * 3 * 2! = 48}$$
$$3\,pipe\!:\ \#J - Curves = 3 * 4 * 3 * 3! = 216$$
$$4\,pipe\!:\ \#J - Curves = 4 * 4 * 3 * 4! = 1152$$
$$5\,pipe\!:\ \#J - Curves = 5 * 4 * 3 * 5! = 7200$$
$$10\,pipe\!:\ \#J - Curves = 10 * 4 * 3 * 10! = 435456000$$
$$20\,pipe\!:\ \#J - Curves = 20 * 4 * 3 * 20! = 5.83896E + 20$$



The results of the J-Curve analysis are summarized in the table below.

Table 2: J-Curve Analysis

Segment	Optimum Pressure (Psig)	Optimum Diameters (Inches)	TAC per MCF	Total Annual Cost (Millions)
1	750	18 & 18	$ 0.631	$ 66
2	850	18 & 18	$ 0.616	$ 65
Overall	850	18 & 18	$ 0.616	$ 65

Mathematical Model Analysis on Two-Segment Pipeline

The two-segment pipeline was then input into the mathematical model, and the results could now be directly compared to the results found using the J-Curve analysis. These results were tabulated below:

Table 3: Non-Linear Model – 2 Pipe Network

	Pipe 1	Pipe 2
Pipe Diameter (in)	22	22
Compressor Work (hp)	10,740	0
Pressure Drop (psi)	1,830	1,490
TAC Model	$ 0.596	
TAC J-Curves	$ 0.616	

The results of this comparison demonstrate the power of using mathematical programming to optimize a pipeline network. Instead of running 432 simulations and locating the minimums during the J-Curve, which took many hours to perform; the model took a few minutes to input the specifications of the system and only seconds to optimize. Also, the non-linear model was able to analyze a wider range of diameters and pressures which would have increased the amount of workload involved in the J-Curve analysis. Furthermore, the model was able to analyze each supplier and consumer node and determine whether or not a compressor should go there or not. Thus, in the optimal solution provided by the model is the two segments having pipe diameter of 22 NPS and only one compressor instead of the two suggested. The ability to choose exactly how many compressors, and where they should be located is something that is unique to the model. The model also saves the user money, since the TAC per Mcf is now only $0.596 compared to the TAC found in the J-Curve method of $0.616.

In conclusion, the mathematical programming model was proven to be a far superior method of optimization of pipeline networks when compared to the J-Curve analysis method. This method was proven better because it: found a lower TAC value, was able to analyze a wider range of pipe sizes and compressor locations, and saved the user many hours of valuable time. In fact, if the pipeline network was a ramified system with many consumers, suppliers and pipeline branches it would take an unrealistic amount of time to apply the J-Curve method appropriately to the system as proven in Appendix B. Therefore, this comparison has provided evidence that the non-linear mathematical model is a superior method of optimization and research should be continued with this model.

The New Mathematical Programming Logic

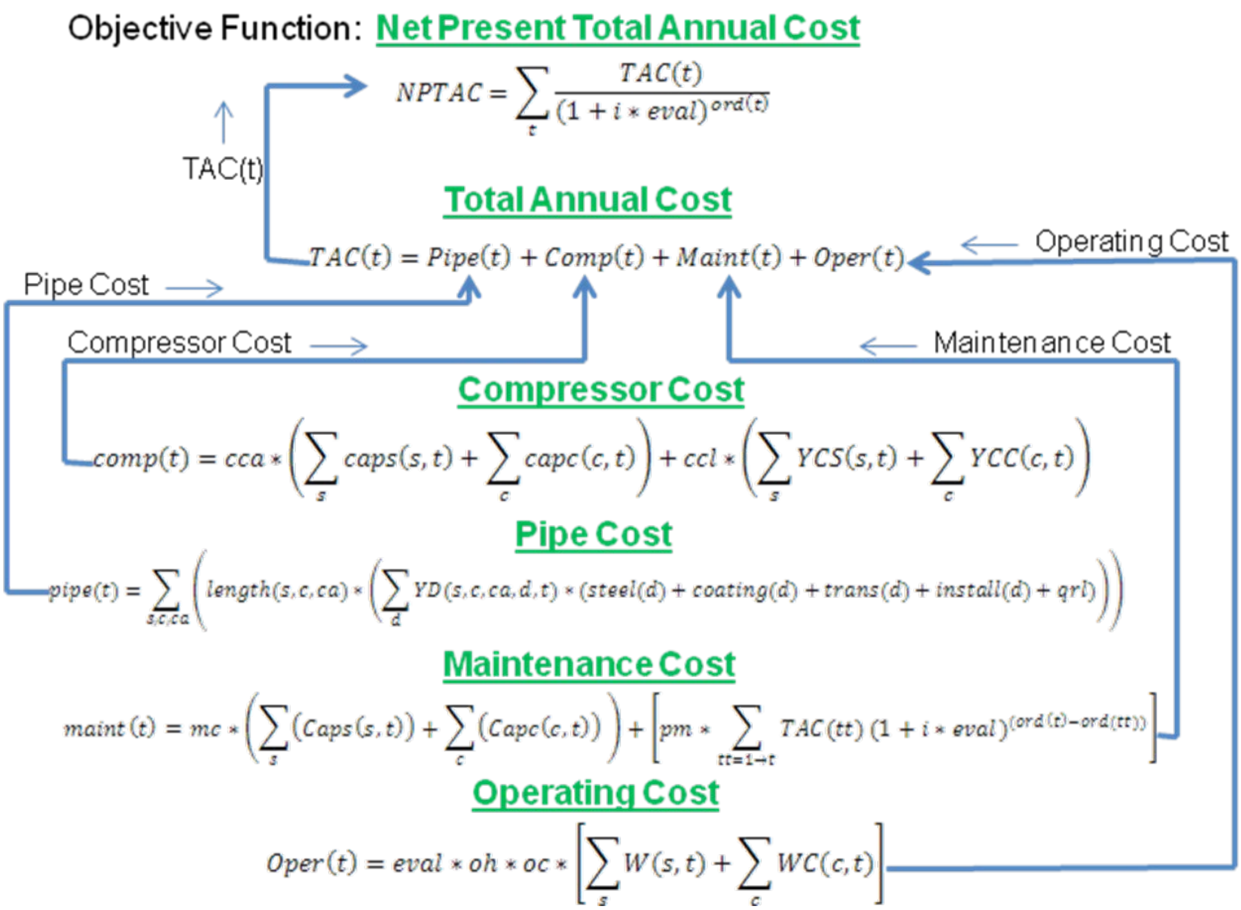
So how does this model work? In a nutshell, the linear model generates discrete pressures. The model then optimizes the entire system based on these discrete pressures. The optimum pipe diameters, compressor locations, compressor sizes, and compressor installation times are given. The results are then applied to the nonlinear model which does not use discrete pressures. This model minimizes net present total annual cost to find more precise pipe diameters, compressor locations, compressor sizes, and compressor installation times.

Some of the inputs for the mathematical program are shown below. Bear in mind that none of this information is information that a designer would not know going into a pipeline design process.

Model	Hydraulics	Economics
Diameter Options	Gas Density	Project Lifetime
Supplier Temperatures	Compressor Efficiency	Operating Cost ($/P*t)
Supplier Pressures	Compressibility Factor	Maintenance Cost ($/hp, %TAC)
Consumer Demands (V/t)	Compressibility Ratio	Operating Hours (hr/yr)
Demand Increase (%/yr)	Heat Capacity	Interest Rate
Min/Max Operating Pressure		Consumer Price ($/V)
Compressor Location Options		Steel Cost(d) ($/L)
Elevations		Coating Cost(d) ($/L)
Pipe Connections		Transportation Cost(d) ($/L)
Distances		Installation Cost(d) ($/L)

A guide to applying a system to the program is shown in Appendix E. Next, the program performs both hydraulic and economic calculations in order to minimize the net present total annual cost. The schematic below shows the flow of economic data.

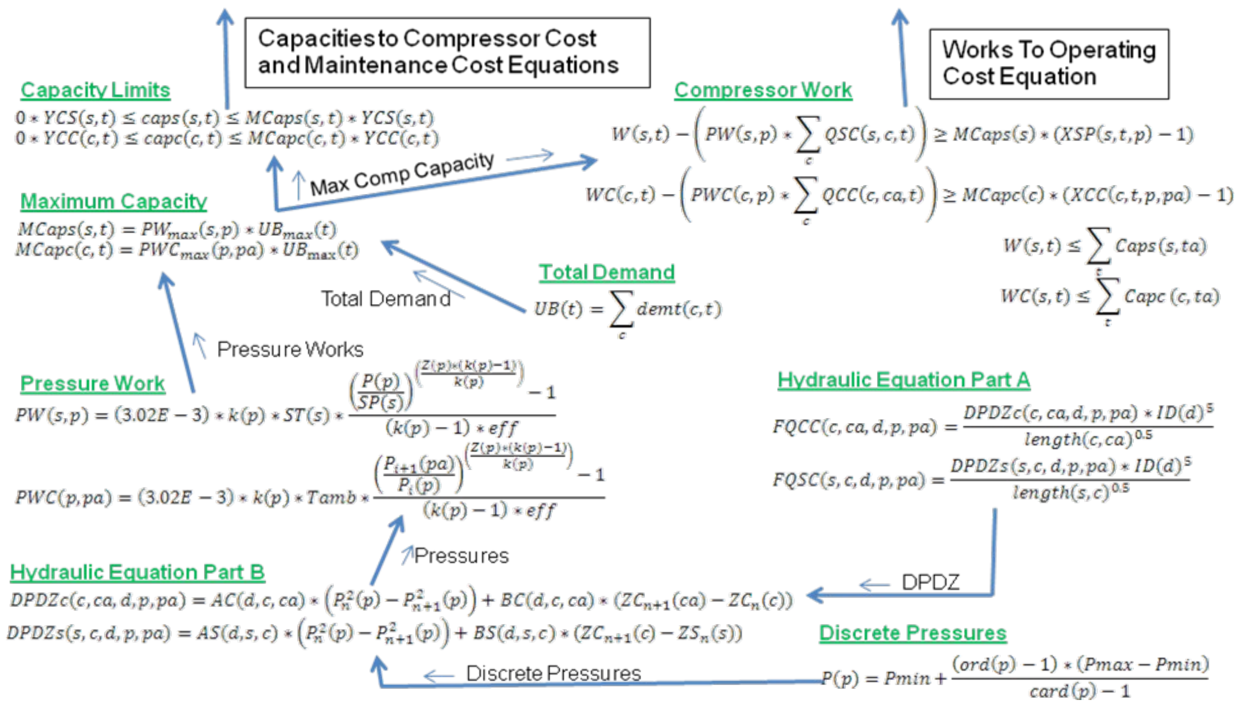
Figure 11: Economic Data in Mathematical Programming



At the top, the objective function (net present total annual cost) is shown. The net present total annual cost is essentially the summation of total annual cost at every time period multiplied by an interest-dependent discount factor. Each total annual cost is the sum of the pipe cost, compressor cost, maintenance cost, and operating cost. The pipe cost is the sum of every pipe length times that diameter's steel cost, coating cost, transportation cost, installation cost, and quadruple random length cost. The compressor cost is an angular compressor cost coefficient times the total compressor capacity, at a given time, plus a linear compressor cost coefficient times the total number of compressors at that time. The maintenance cost is the sum of the pipeline maintenance cost and the compressor maintenance cost. The pipeline maintenance cost is a user-defined coefficient times the previous time period's total annual cost. The compressor maintenance cost is a user-defined coefficient times the total compressor capacity at that point in time. The operating cost is essentially a user-defined coefficient times the total work of all compressors at that time multiplied with the total operating hours in a year.

The compressors works and capacities come from hydraulic equations in both the linear and nonlinear models. The linear hydraulic calculations are shown below.

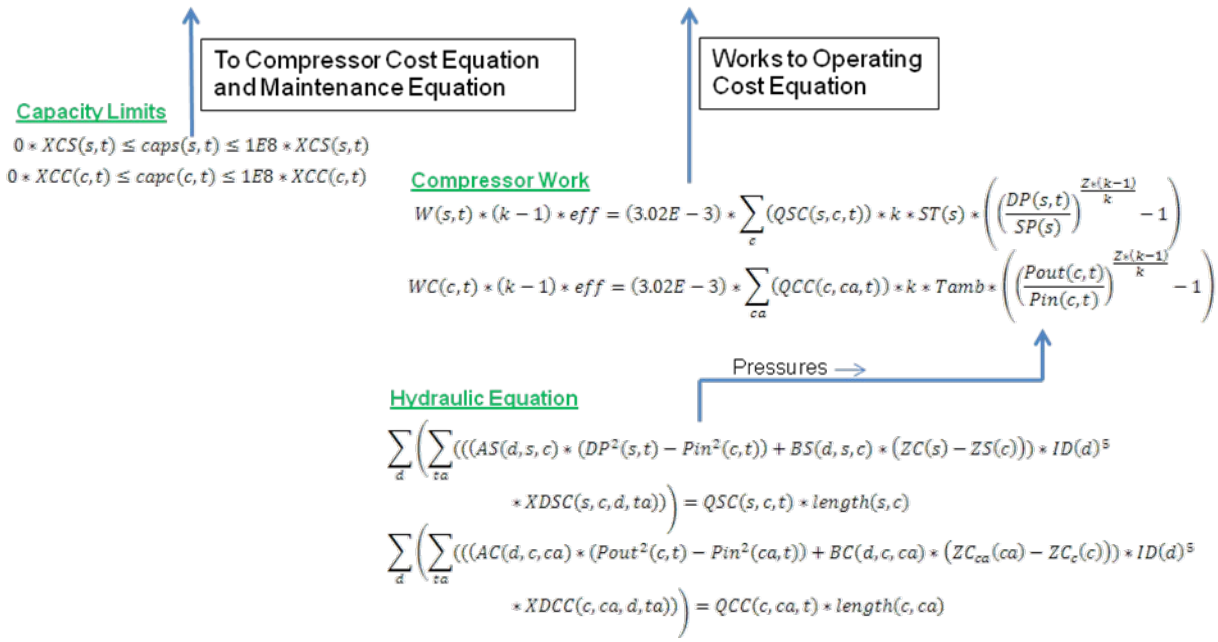
Figure 12: Linear Hydraulic Calculations in Model



This looks rather confusing. So let's walk through it starting at the bottom. First, discrete pressures are generated. These discrete pressures are generated such that they are equally spaced between the minimum and maximum operating pressures. These discrete pressures are then plugged into the proposed model equation evaluated earlier. The resulting pressures are then used to find pressure work. These pressure works are then combined with the total demand to determine a hydraulic maximum for each compressor capacity. These maximum capacities are input as limits for the actual compressor capacity and actual compressor work. The actual compressor capacity and actual compressor work are then inserted into the economic equations.

The nonlinear calculations are a little simpler and are shown below. They are simpler because they do not have to deal with the discrete pressures.

Figure 13: Linear Hydraulic Calculations in Model



Pressures are generated by the model equation discussed earlier. These pressures are then used to calculate compressor works which are then sent to the economics equations. The compressor capacities are maximized here not by a maximum capacity but by a large number.

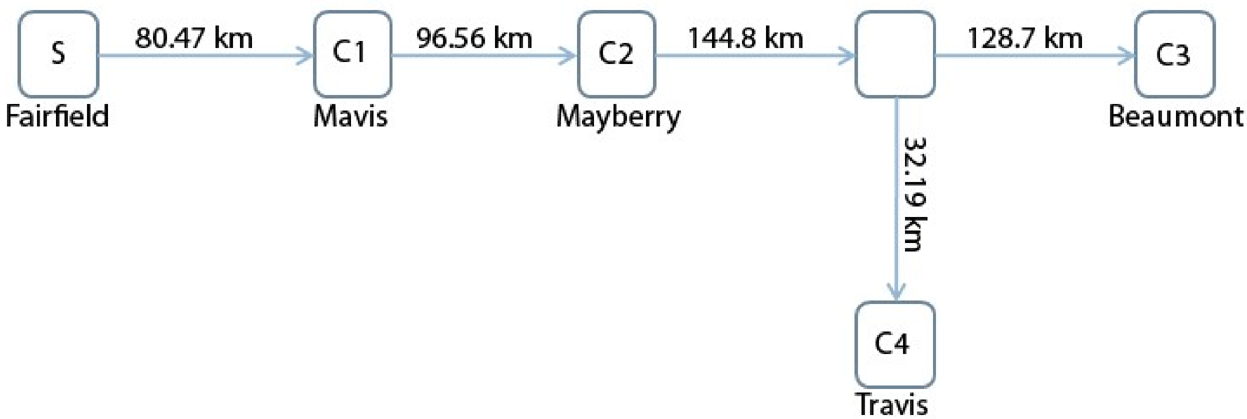
The outputs of the model are shown below.

Physical	**Economics**
Pipe Locations	Net Present Value
Pipe Diameters	Net Present Total Annual Cost
Demand at Each Period	Total Annual Cost at Each Period
Flowrates	Fixed Capital Investment
Inlet and Outlet Pressures	Revenue
Compressor Locations	Operating Cost
Compressor Capacities	Pipe Cost
	Compressor Cost
	Maintenance Cost
	Penalties

Applying the Model

In order to test the mathematical model, it was applied to the following case study. This case study is from Gas Pipeline Hydraulics by Menon et. al. and has been modified to account for the variations of natural gas flowrates between the high demand winter months and lower demand summer months as well as an annual demand increase of 10%. The project lifetime is taken to be 8 years.

Figure 14: Case Study on Ramified Network



Fairfield	
Supply P (kPa)	3548.7
Supply T (°R)	529.67
MinOP (kPa)	10050.5
MaxOP (kPa)	4200
Elevation (km)	0.185928

	Mavis	**Mayberry**	**Split**	**Beaumont**	**Travis**
Initial Demand (Mcmd)	283.17	566.34	0	2831.7	1699
Price ($/m^3)	0.32	0.33	0	0.3	0.3
Elevation (km)	0.56376	0.54864	0.2286	0.10668	0.12816

It is important to note that this information is not an uncommon collection of known data when entering into a pipeline design process. A natural gas supplier located in Fairfield is to supply natural gas to four consumers: Mavis, Mayberry, Beaumont, and Travis. Consumer initial demands, costs, and elevations are known. The split point in the schematic is programmed as a zero-demand consumer, but it is still included because its elevation and location are very important to the hydraulics of the system.

This model is first applied to the linear model. The linear model finds optimum conditions which minimize the net present total annual cost at the end of the project lifetime. The linear model finds this optimum using a user-defined number of discrete pressures. The optimums found by the linear model are then applied to the nonlinear model. The nonlinear model does not use discrete pressures. This allows the nonlinear model to find more precise results as it is able to access the full range of pressures instead of just the discrete ones generated by the linear model. The results of the programming analysis are shown below.

Table 4: Non-Graphical Results

Case Study Results	
Pipe 1 ID (in.)	22
Pipe 2 ID (in.)	22
Pipe 3 ID (in.)	22
Pipe 4 ID (in.)	18
Pipe 5 ID (in.)	12
NPTAC ($)	243,706,100
Pipe Cost ($)	185,720,700
Supplier Compressor Capacity (hp)	22,929.16
Consumer1 Compressor Capacity (hp)	13,365.09
Consumer2 Compressor Capacity (hp)	13,293.76
Consumer3 Compressor Capacity (hp)	8,439.168

Figure 15: 8 Year Economic Analysis for Case Study

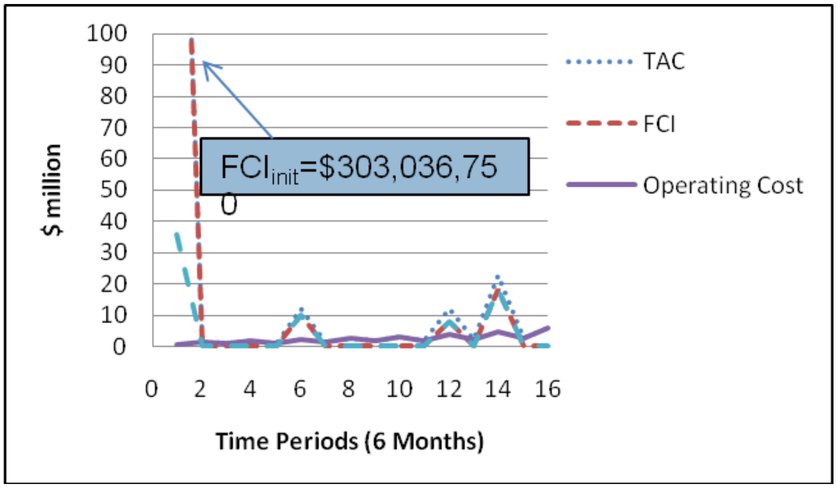
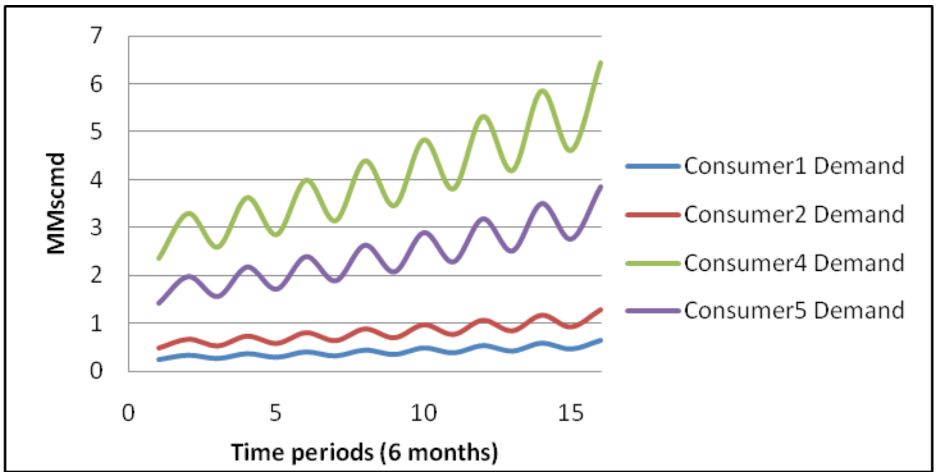


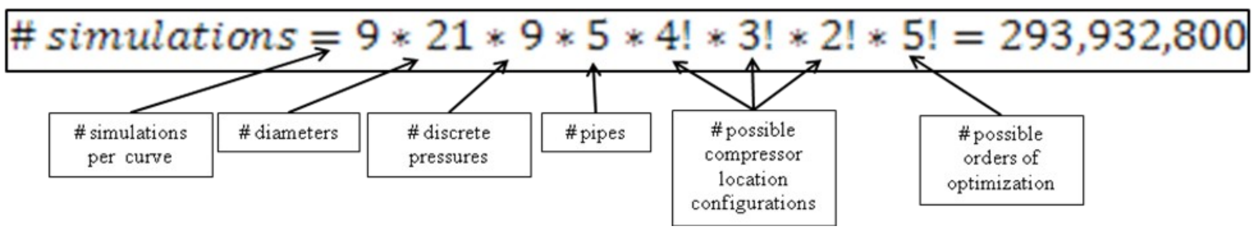
Figure 16: 8 Year Consumer Demand Functions



It was first observed that the diameters decrease as the pipeline progresses. This makes sense because as flowrate (demand) decreases, so should the pipe size. Moving on to the economics graph, a few observations can be made. First, it is clear that most of the investment occurs at the beginning of the project. This is because all of the pipes as well as the largest compressor are installed initially. Next, one can essentially see a timeline of compressor installations. Then, a compressor is installed at 3, 6, and 7 years. Each peak corresponds to a compressor size given in the table. The height of the peaks in relation to one another tells the analyzer which compressor is which. Next, it is apparent that the operating cost increases as the total compressor work increases. The largest jumps can be seen when new compressors are installed. Also, the total annual cost is essentially the fixed capital investment plus the operating cost. This is observed graphically by the

TAC line. The second graph shows consumer demand. One can observe that both the seasonal demand variations as well as the 10% annual demand increase with time.

So this raises the question, "Why should the J-Curve method not be used to solve this problem?" The fact of the matter is that the J-Curve analysis would take a remarkable amount of time and labor. This can be seen below.



In order to fully optimize the system to the extent that the linear mathematical program does, it would require 293,932,800 simulations. This assumes 9 simulations per curve, 21 pipe diameter options, 9 discrete pressures, 5 pipes, 4!*3!*2! possible compressor location configurations, and 5! possible orders of optimization. This would take one person working 24 hours a day and 7 days a week 2769 years. If this person worked the standard 40-hour work week, it would take 11,776 years. In order to accomplish this design in 6 months, it would require 23,552 employees. Even if these employees worked for the Oklahoma minimum wage, this would cost $153,088,000. Even still, the results would be inaccurate because the J-Curve method implements discrete pressures. To obtain a true optimum, it would require an infinite number of discrete pressures.

Recommendations

Although much advancement has been made with the research and expansion of the pipeline optimization model, it is not yet a finished product. The future work on this model will consist of adding uncertainty to the model. Since the model uses large amounts of forecasted information, it is important to add uncertainty to the model to make the results compare more to a real system's behavior. Furthermore, many more cost function can be added and updated to the model making it a more robust tool. This can be done with ease by working with an industrial partner that knows all the costs associated with a natural gas pipeline. Another expansion will be adding bursting pressure calculations to the model in order to determine the appropriate pipe thickness that will be used in the network. This will be a very useful function because the thickness will not have to be calculated by hand and it is something that greatly affects the cost of the pipe. Finally, a combination of the linear and non-linear models will be performed, and in order to streamline the optimization process thus making the program more user friendly.

Once these adjustments are made on the model, the potential of this tool is endless. It can be couple with a GAMS data exchanger (GDX) file that could tie in a user-interface with the program. This will make the model easier to use and a better tool for industry. One interface that would be possible is Microsoft Excel. This would enable unit converters to be tied into the model so that the user can use whatever units on the pipeline that they prefer. Even though the program is not complete, the results of this report have proven the potential value of this tool, and therefore continued research is absolutely essential. Once these additions are made, this model will be a very powerful tool.

Conclusion

The natural gas industry is always looking for ways to improve their pipeline efficiencies and to improve the bottom line. The conventional methods researched and examined in this report have been proven to produce large amounts of costly error and are also very time consuming. The hydraulic equations that are being used in the industry are producing error that puts millions of dollars of natural gas in the hands of uncertainty every year. Furthermore, J-Curve economics are too time-consuming for complicated ramified networks if they are going to be appropriately applied. Therefore, a need for a better tool to calculate the hydraulics and economics of natural gas pipelines is absolutely necessary. The mathematical programming model used for pipeline optimization is this tool. With continued research and expansion of this model, it could become invaluable to the natural gas industry.

Appendix A

A Comprehensive Study of Various Natural Gas Hydraulic Equations

The following is a list of some correlations that are used in the natural gas industry (Menon):

1. Colebrook-White equation
2. Modified Colebrook-White equation
3. AGA equation
4. Panhandle
    i. A equation
    ii. B equation
5. Weymouth equation
6. IGT equation
7. Spitzglass equation
8. Mueller equation
9. Fritzsche equation

In this report, these equations are investigated at different operating conditions. The output is then compared to the output from Pro/II simulations with the same conditions. Pro/II produces an accurate answer for the pressure drop in the pipe. It utilizes the Beggs, Brill, and Moody method which takes into account the different horizontal flow regimes including segregated, intermittent, and distributed. Also, a General Algebraic Modeling System (GAMS) program was investigated under the same conditions as the correlation above. This investigation was conducted in order to determine the amount of error involved while using these correlations and the mathematical programming model when compared to a Pro/II simulation. The results of this study will indicate how accurate these equations are compared to Pro/II simulation and compared to the mathematical programming model.

Pipeline Hydraulic Fundamentals

For fluids flowing in a pipeline between two points (A and B), the energy balance is subject to the following equation known as the Bernoulli's equation[2]:

$$Z_A + \frac{P_A}{\gamma} + \frac{V_A{}^2}{2g} + H_p = Z_B + \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + h_f \tag{A1}$$

where

Z_A = Elevation at point A

Z_B = elevation at point B

P_A = pressure at point A

P_B = pressure at point B

V_A = velocity at point A

V_B = velocity at point B

g = gravity constant

γ = gravity times the density of the fluid

H_p = the equivalent head added to the fluid by a compressor at point A

H_f = the total frictional pressure loss between points A and B

The equations studied in this report are all derived from the equation above. Each equation takes into account different simplifications and gas laws. The formulas all relate the properties of the gas to the flow rate, pipe diameter and length, and the pressure in the pipe. This report will show the accuracy of each equation used and under what circumstances the formulas are valid.

The general flow equation is the following[2]:

$$Q = 77.54\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - P_2^2}{GT_f LZf}\right)^{0.5} D^{2.5} \quad \text{(USCS units)} \tag{A2}$$

Where

Q = gas flow rate, at standard conditions, ft^3/day (SCFD)

f = friction factor, dimensionless

P_b = base pressure, psia

T_b = base temperature, °R

P_1 = upstream pressure, psia

P_2 = downstream pressure, psia

G = gas gravity (air = 1.00)

T_f = average gas flowing temperature, °R

L = pipe segment length, mi

Z = gas compressibility factor at the flowing temperature, dimensionless

D = pipe inside diameter, in

This equation is for steady state isothermal flow for a gas in a pipe. It is the basic equations that many of the following equations are derived from. Often the equation uses a specific transmission factor (F):

$$F = \frac{2}{\sqrt{f}} \tag{A3}$$

where f is the Darcy friction factor.

It can also be defined by:

$$F = \frac{1}{\sqrt{f_f}} \tag{A4}$$

where f_f is the Fanning friction factor. Various versions of the transmission factor are used in General Flow Equation to produce many of the equations examined in this report.

Results

Colebrook Equation

The Colebrook Equation, also known as the Colebrook-White Equation, introduces the following transmission factor into the General Flow Equation[2]:

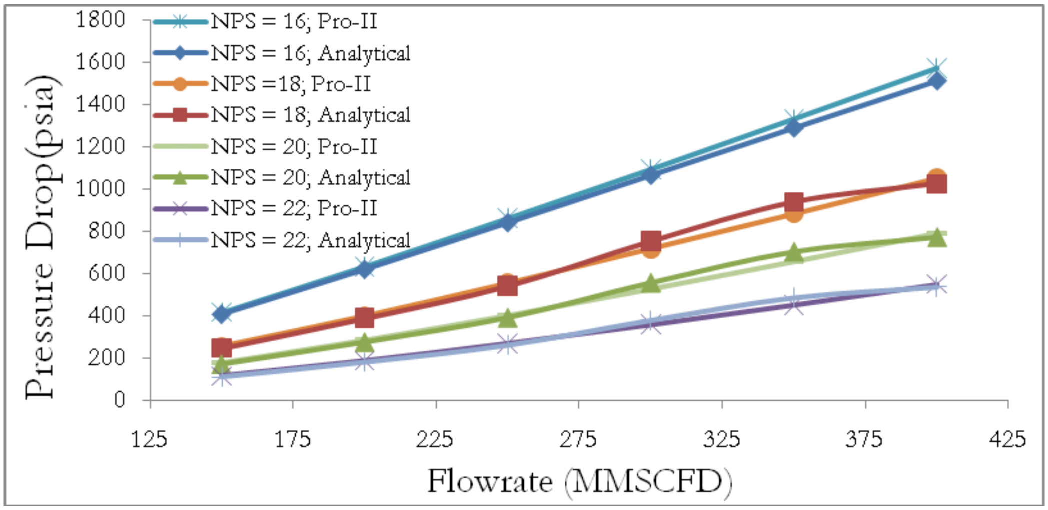
$$\frac{1}{\sqrt{f}} = -2\,\mathrm{Log}_{10}\left(\frac{e}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right) \quad \text{for } Re > 4000 \tag{A5}$$

The assumption of turbulent flow in a smooth pipe reduces the above equation to[2]:

$$\frac{1}{\sqrt{f}} = -2\,\mathrm{Log}_{10}\left(\frac{2.51}{Re\sqrt{f}}\right) \tag{A6}$$

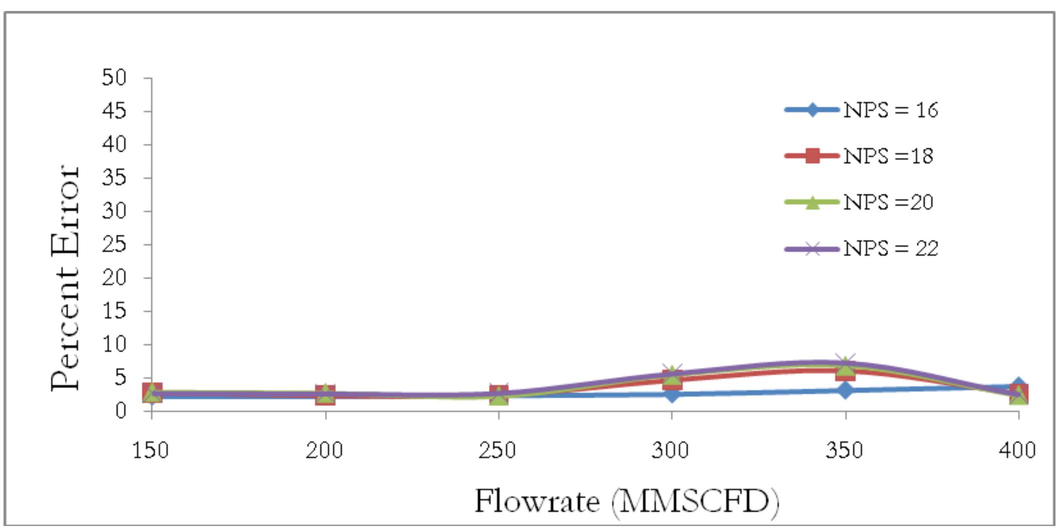
This was plugged into the General Flow Equation to obtain the values for the pressure drop at various flow rates. The graph below was for a pipeline that does not experience a change in elevation.

Figure A1: Pressure Drop Comparison -- Colebrook & Pro/II



The values obtained using the Colebrook Equation were compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

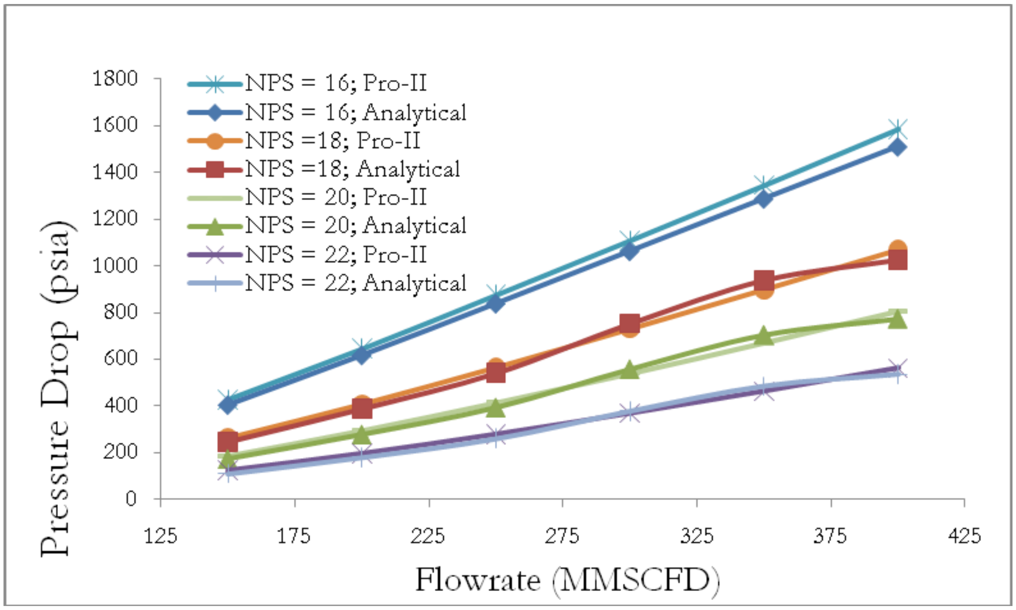
Figure A2: Error Anaylsis -- Colebrook & Pro/II



The result shows that the Colebrook Equation has fairly accurate results for lower flowrates (150-250 MMSCFD); however, larger error exists above a 250 MMSCFD flow rate.

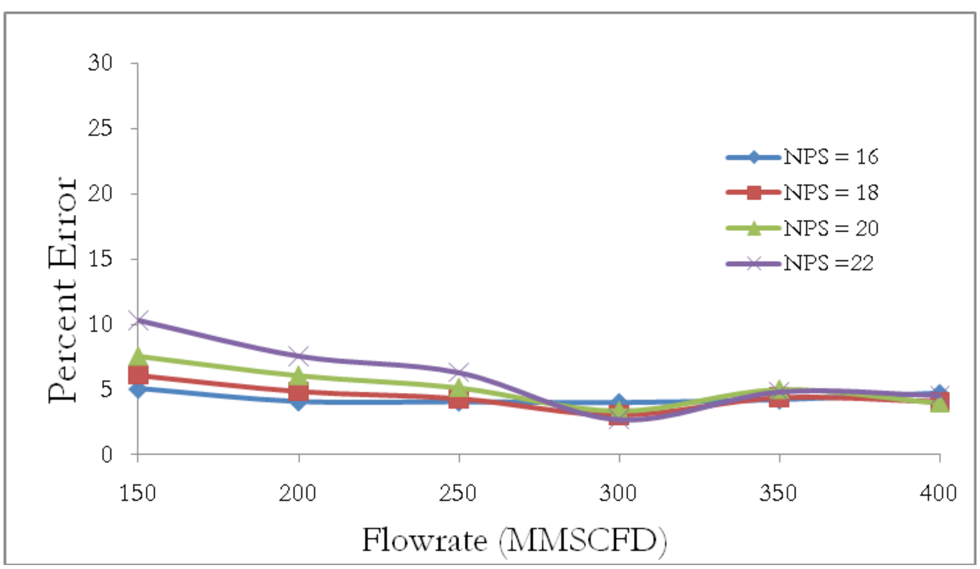
Next, an elevation change of 0.15 km was applied to the equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A3: Pressure Drop with Elevation Change -- Colebrook & Pro/II



This equation was then compared to Pro/II simulations in order to determine the accuracy of the equations with pressure drop being taken into account.

Figure A4: Error Anaylsis with Elevation -- Colebrook & Pro/II



This indicates that the elevation change produced a similar amount of error. However, at low flowrates and large pipe diameters, the Colebrook Equation produced error around 10% which is about three times the amount of error than when there was no elevation change.

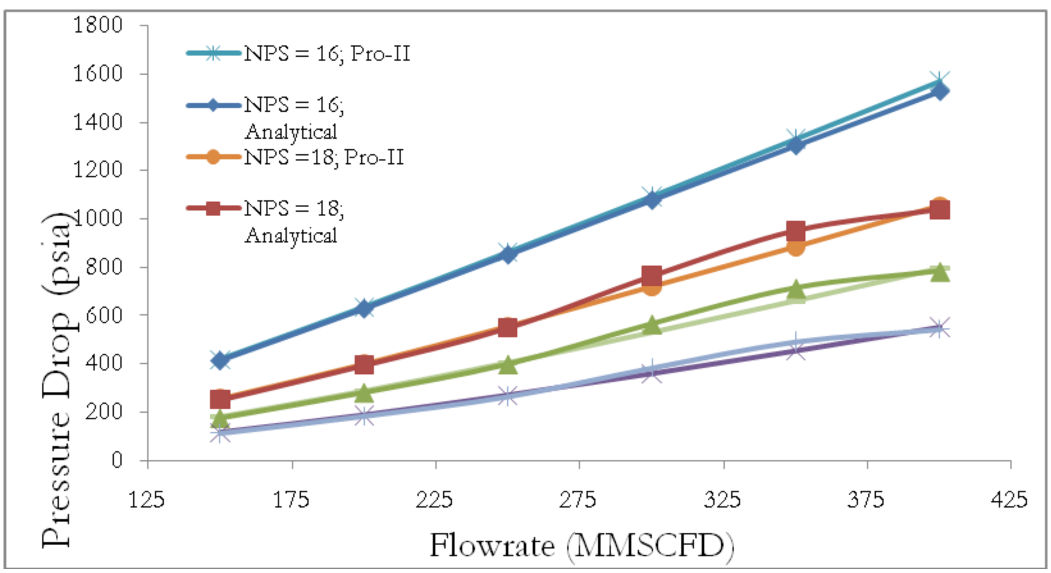
Modified Colebrook Equation

The Modified Colebrook Equation incorporates a slightly different transmission factor into the General Flow Equation[2].

$$\frac{1}{\sqrt{f}} = -2\,\mathrm{Log}_{10}\left(\frac{e}{3.7D} + \frac{2.825}{Re\sqrt{f}}\right) \tag{A7}$$

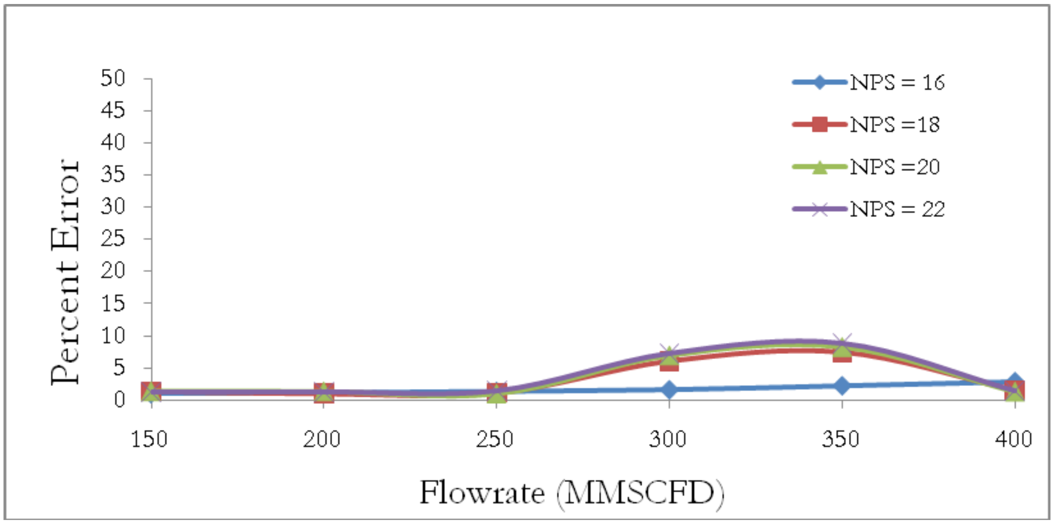
This was also inserted into the General Flow Equation to obtain the values for the pressure drop at various flow rates.

Figure A5: Pressure Drop Comparison -- Modified Colebrook & Pro/II



The values obtained using the Modified Colebrook Equation were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

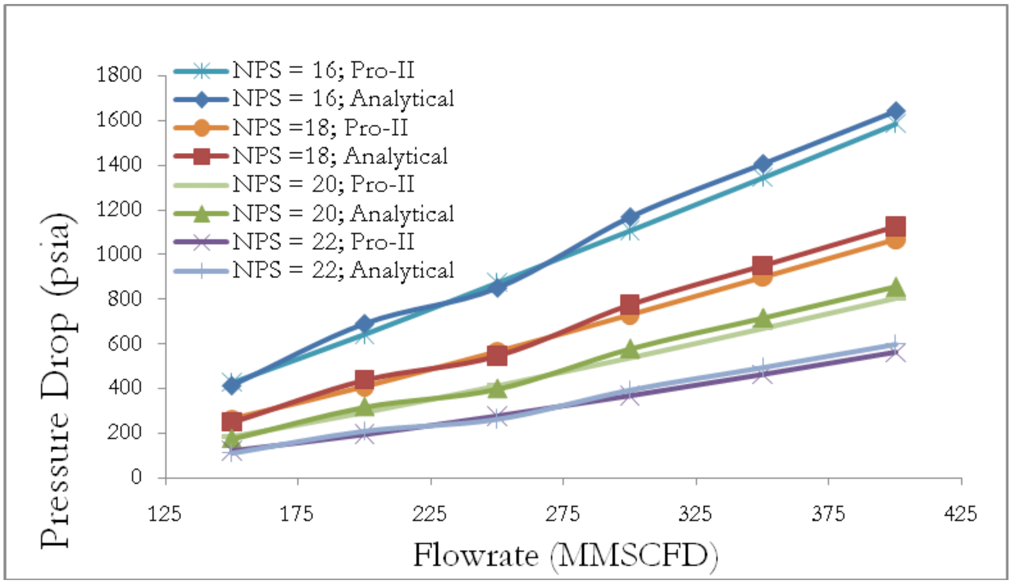
Figure A6: Error Anaylsis -- Colebrook & Pro/II



The Modified Colebrook Equation has very low error for flowrates less than 250 MMSCFD; however, the amount of error becomes quite significant for flowrates above that point.

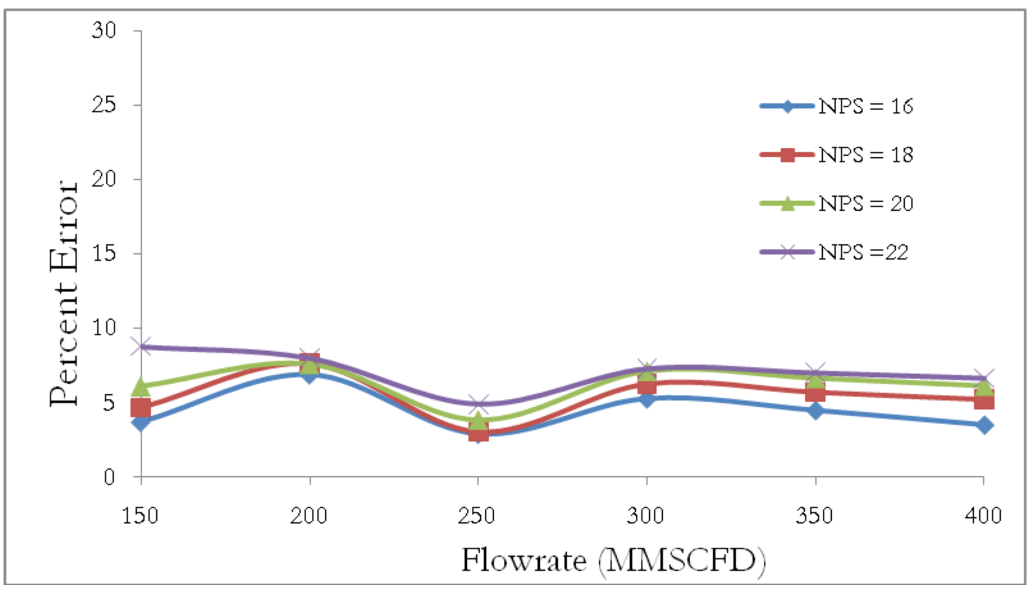
Again, an elevation change of 0.15 km was applied to the equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A7: Pressure Drop Comparison with Elevation -- Modified Colebrook & Pro/II



This equation was also compared to Pro/II simulations in order to determine the accuracy of the equations with pressure drop being taken into account.

Figure A8: Error Anaylsis with Elevation – Modified Colebrook & Pro/II



This equation has a much larger (5-10%) amount of error when compared to a pipe with no elevation change.

American Gas Association (AGA) Equation

Similar to the Colebrook Equation, the AGA Equation uses a slightly modified transmission factor in order to obtain a value for the pressure drop using the General Flow Equation. The transmission value for the AGA equation is the following[2]:

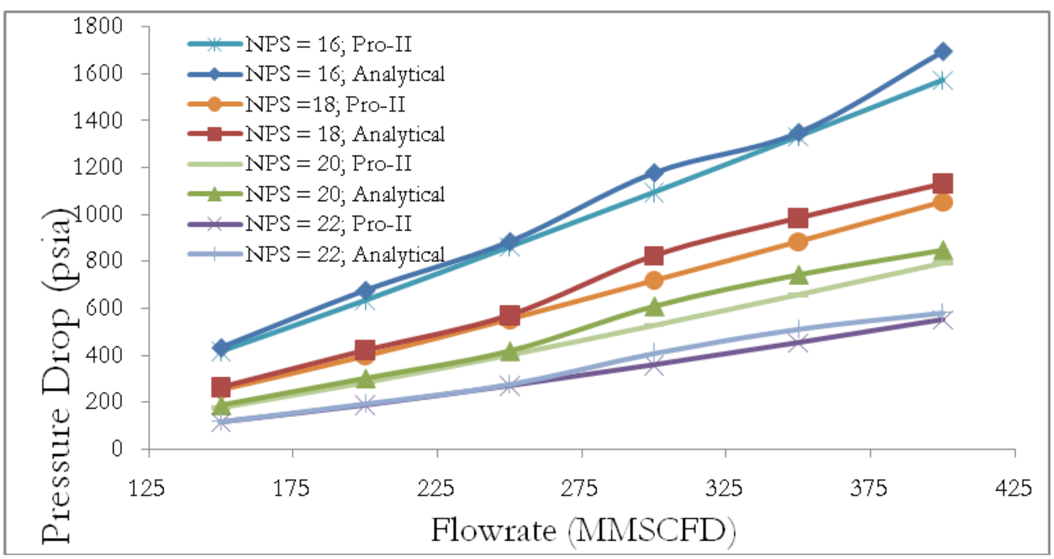
$$F = 4\,\mathrm{Log}_{10}\left(\frac{3.7D}{e}\right) \tag{A8}$$

This equation is also known as the Von Karman equation for rough pipe flow. If turbulent flow is assumed, then the equation reduces to[2]:

$$F = 4D_f\,\mathrm{Log}_{10}\left(\frac{Re}{1.4125F_t}\right) \tag{A9}$$

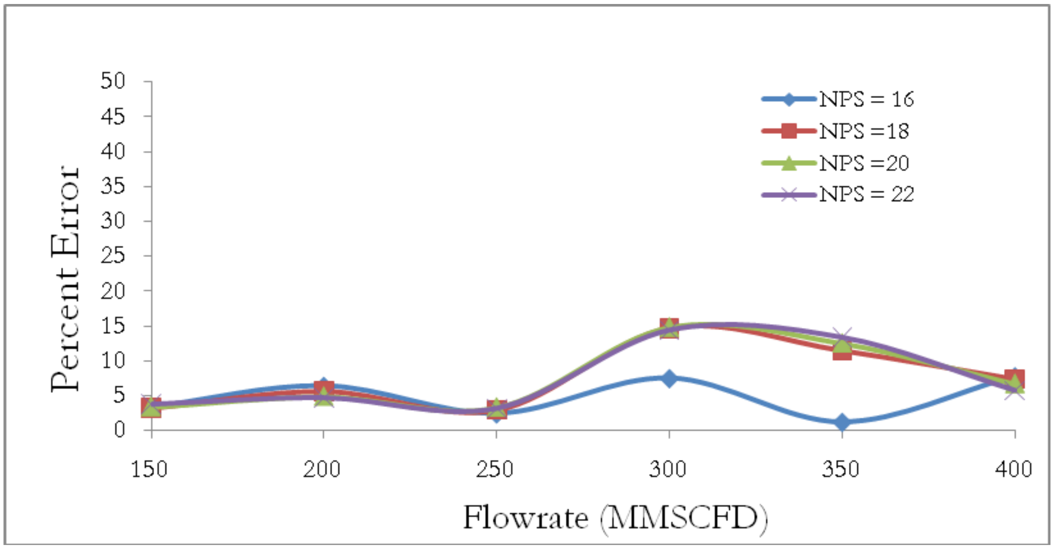
where D_f is the pipe drag factor. A table of D_f for various pipe materials are used in order to determine the appropriate value depending on the circumstances. An assumption of bare steel pipe with extremely low bend produced the values for the pressure drop in the graph below.

Figure A9: Pressure Drop Comparison -- AGA & Pro/II



The values obtained using the AGA Equation were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

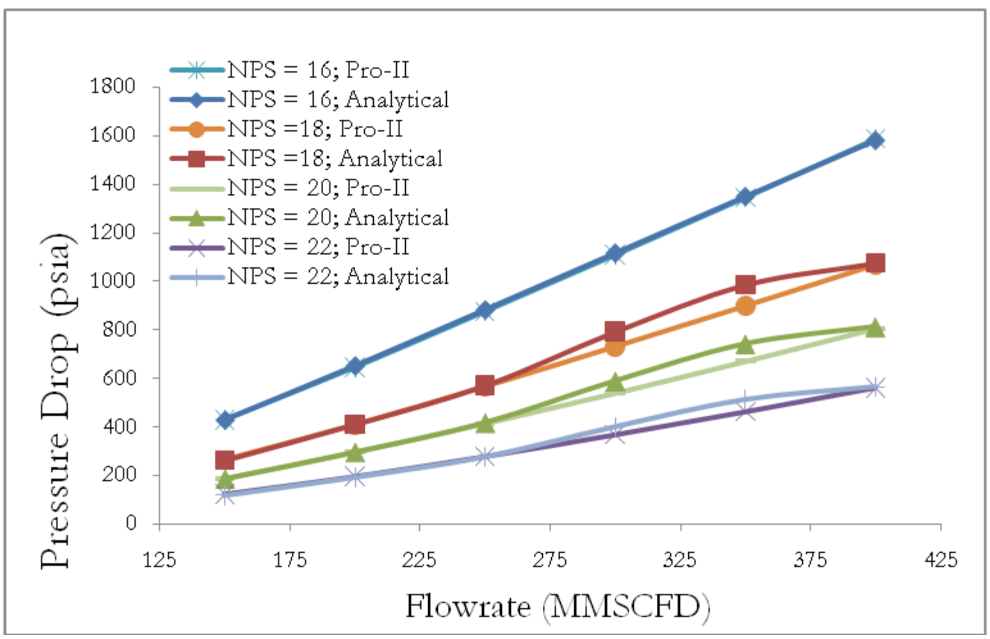
Figure A10: Error Anaylsis -- AGA & Pro/II



This indicates that the AGA Equation produces significant error at high flowrates and large pipe diameters.

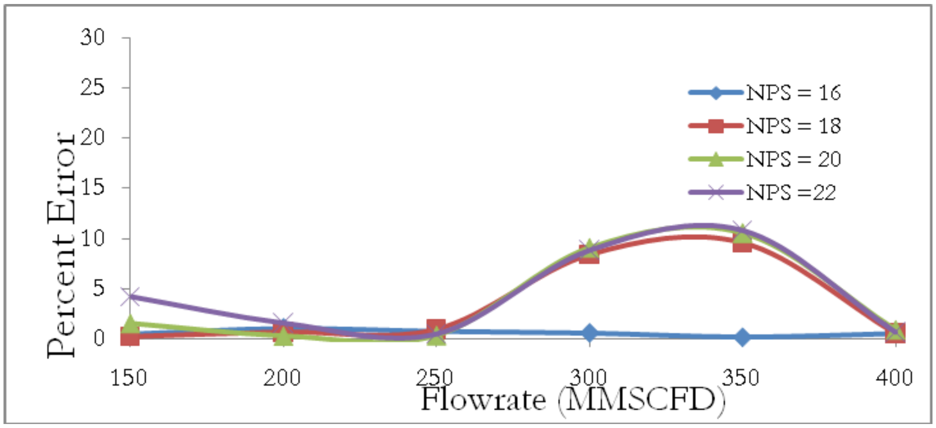
Again, an elevation change of 0.15 km was applied to the equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A11: Pressure Drop Comparison with Elevation -- AGA & Pro/II



The values obtained using the AGA Equation with a 0.15 km elevation change were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A12: Error Anaylsis with Elevation – AGA & Pro/II



This graph shows that at smaller diameter pipes, this equation remains accurate for the various flow rates. However, larger diameter pipes produce error of about 10% for flowrates greater than 250 MMSCFD.

Weymouth Equation

The Weymouth Equation is mainly used for systems with high pressure, high flow rates, and large diameter pipe. Therefore, this equation is used mainly for transmission lines and not for gathering and distribution pipelines. The transmission factor used for the in USCS is[2]:

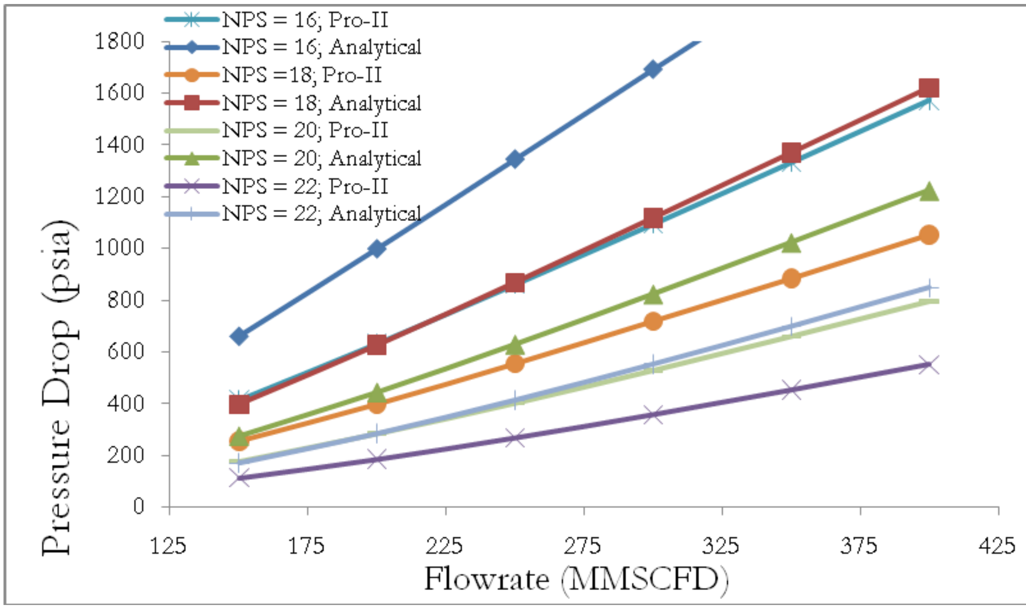
$$F = 11.18(D)^{1/6} \tag{A10}$$

Once this transmission factor is plugged into the General Flow Equation it reduced to Weymouth Equation shown below[2]:

$$Q = 433.5E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^s P_2^2}{GT_f L_e Z}\right)^{0.5} D^{2.667} \tag{A11}$$

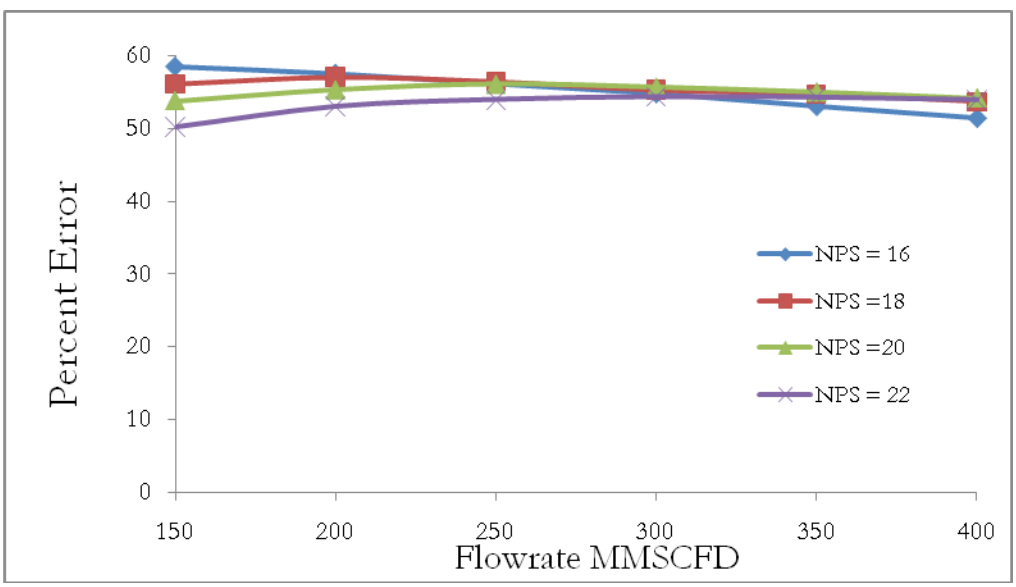
where E is the pipeline efficiency, a decimal value less than 1.0. This equation was also implemented to determine a pressure drop in a pipe segment under various flow rates.

Figure A13: Pressure Drop Comparison -- Weymouth & Pro/II



Just as in previous equations, the values obtained using the Weymouth Equation were compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flowrates.

Figure A14: Error Anaylsis -- Weymouth & Pro/II

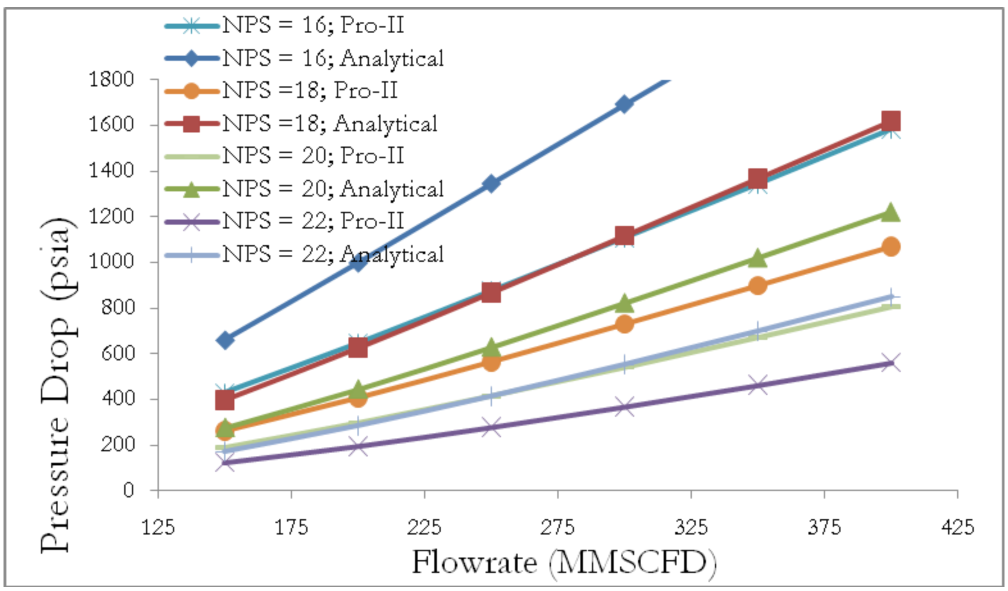


This equation is the first equation to show an extreme amount of error. For all diameters entered, the percent error was around 50-60% compared to the Pro/II data for all the various flow rates.

The error tends to increase with flowrates which suggest that the Weymouth Equation is only valid for extremely high flowrates and pressures.

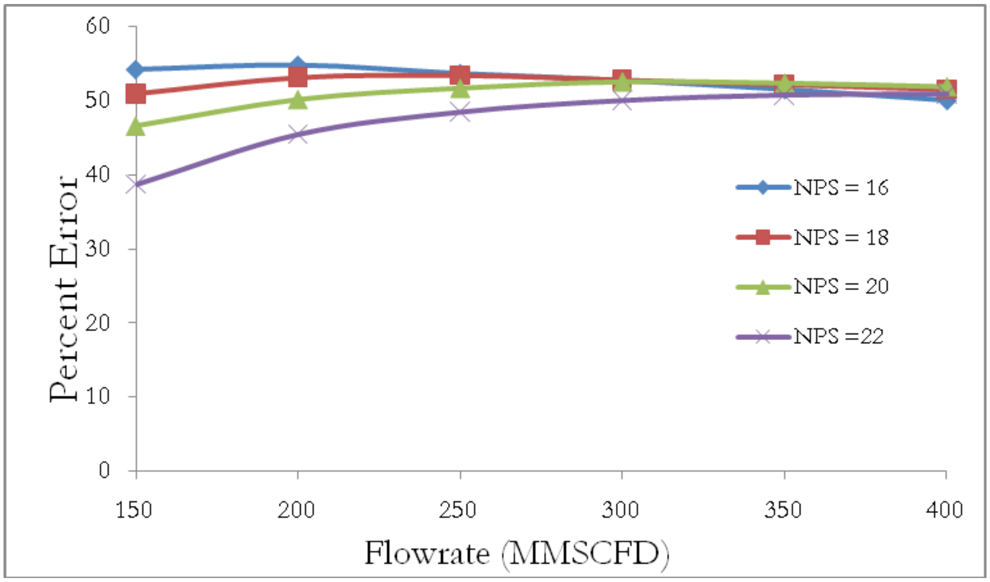
For consistency, an elevation change of 0.15 km was applied to the equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A15: Pressure Drop Comparison with Elevation -- Weymouth & Pro/II



The values obtained using the Weymouth Equation, with a 0.15 km elevation change, were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A16: Error Anaylsis with Elevation – Weymouth & Pro/II



This graph is similar to amount of error produced with the Weymouth Equation without an elevation change. Although, the major difference is that for a larger diameter pipe, the error was 20% less the 150 MMSCFD flow rate.

Panhandle Equation

The Panhandle A Equation incorporates the gas properties associated with natural gas into the general energy balance equation. It has an efficiency factor for the Reynolds number within a range of 5 to 11 million. The roughness of the pipe is not directly inputted into the equation; however, the efficiency factor is built into the equation to take into account inefficiencies caused by the roughness of the pipe. This equation introduces the following transmission factor into the General Flow Equation[2]:

$$F = 7.2111E\left(\frac{QG}{D}\right)^{0.07305} \quad \text{(USCS)} \tag{A12}$$

The transmission factor is equal to two divided by the square root of the friction factor. This reduces the equation to the Panhandle A equation for USCS units is[2]:

$$Q = 435.87E\left(\frac{T_b}{P_b}\right)^{1.0788}\left(\frac{P_1^2 - e^s P_2^2}{G^{0.8539} T_f L_e Z}\right)^{0.5394} D^{2.6182} \tag{A13}$$

where the variables are defined the same as in previous equations.

The Panhandle B Equation is a slight modification of the original equation which allows the equation to be accurate for a larger Reynolds number range of about 4 to 40 million. It integrates a slightly different transmission factor into the General Flow Equation[2]:

$$F = 16.7E\left(\frac{QG}{D}\right)^{0.01961} \quad \text{(USCS units)} \tag{A14}$$

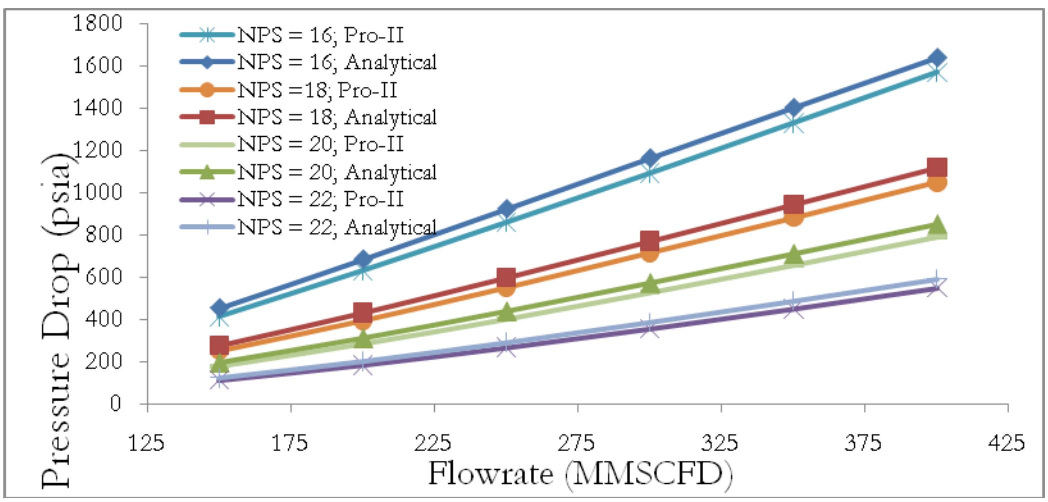
The result of this transmission factor is the Panhandle B equation[2]:

$$Q = 737E\left(\frac{T_b}{P_b}\right)^{1.02}\left(\frac{P_1^2 - e^s P_2^2}{G^{0.961} T_f L_e Z}\right)^{0.51} D^{2.53} \quad \text{(USCS units)} \tag{A15}$$

where the variables are defined the same as in equation A.

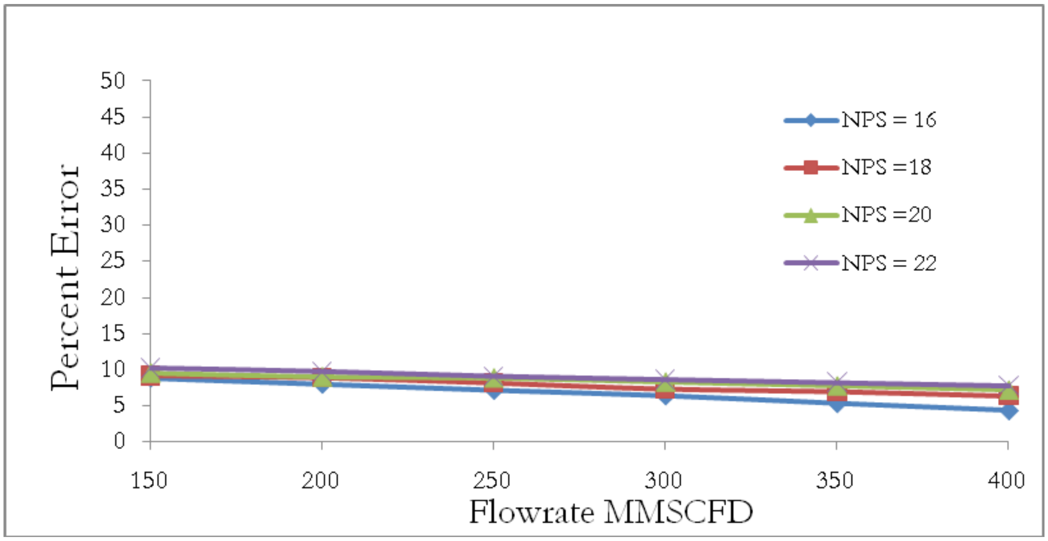
The following chart is the analytical result for pressure drop versus flowrate of natural gas.

Figure A17: Pressure Drop Comparison -- Panhandle & Pro/II



The values obtained using the Panhandle Equations were compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

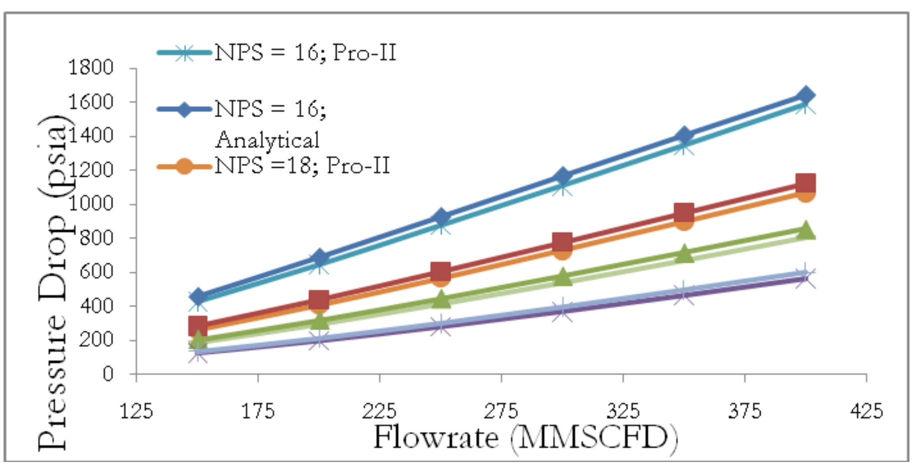
Figure A18: Error Anaylsis -- Panhandle & Pro/II



The chart above shows that the Panhandle equation has an error range from 5-10%. The Panhandle Equation has a higher degree of accuracy for higher flowrates and smaller pipe diameter. The analytical method that was used in this report allowed the program to use the appropriate version of the equation, A or B, depending on the size of the Reynolds number. This method of fusing the two equations together reduced the error for the higher flowrates which require a transmission factor used for the Panhandle B equation.

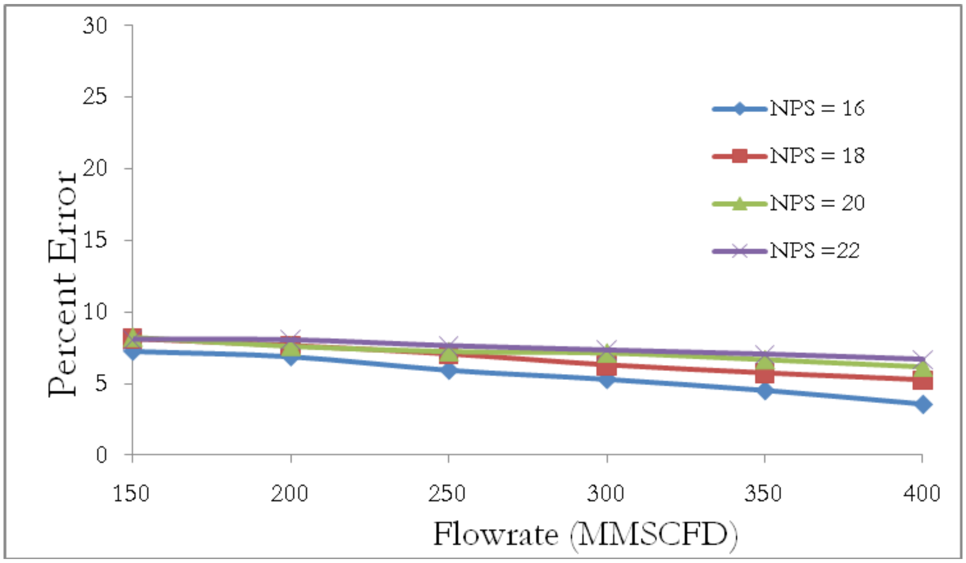
For consistency, an elevation change of 0.15 km was applied to the Panhandle equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A19: Pressure Drop Comparison with Elevation -- Panhandle & Pro/II



The values obtained using the Panhandle Equations with a 0.15 km elevation change were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A20: Error Anaylsis with Elevation –Panhandle & Pro/II



The Panhandle Equation actually produced a more accurate answer when incorporating an elevation change. In fact, there was approximately 2% less error for the 150 MMSCFD flow rate. This implies that the Panhandle Equation is better suited for systems that have elevation changes.

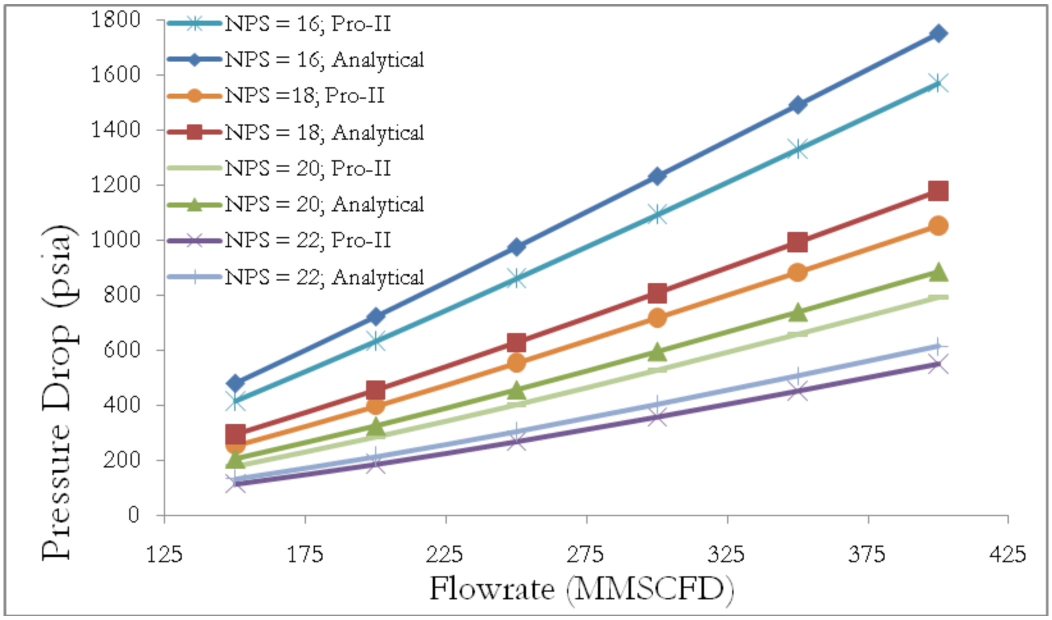
Institute of Gas Technology (IGT) Equation

This equation is similar to the Panhandle and Weymouth equation although slightly different constants are used. The IGT Equation is the following[2]:

$$Q = 136.9E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^s P_2^2}{G^{0.8} T_f L_e \mu^{0.2}}\right)^{0.555} D^{2.667} \quad \text{(USCS units)} \tag{A16}$$

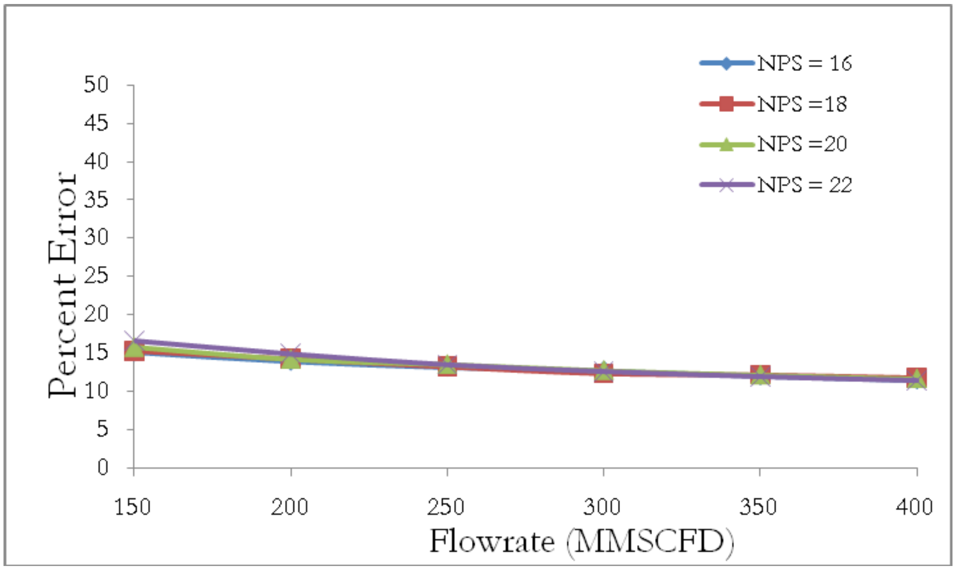
This equation, like the others, was implemented to determine a pressure drop in a pipe segment under various flow rates.

Figure A21: Pressure Drop Comparison -- IGT & Pro/II



The values obtained using the IGT Equation were compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

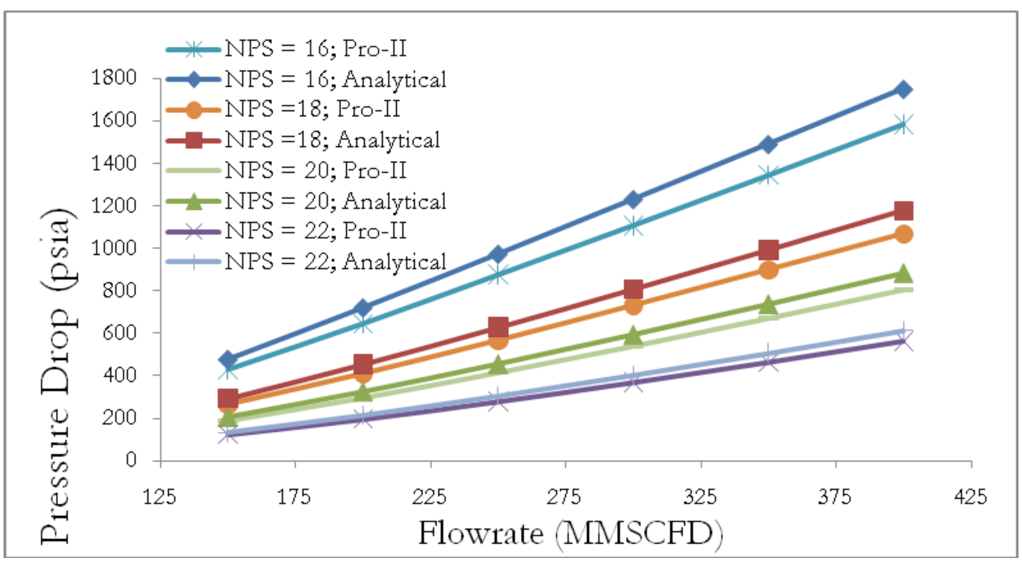
Figure A22: Error Anaylsis -- IGT & Pro/II



The IGT equation produced a significant amount of error under the conditions set in the analytical calculation. An error trend that can be deducted from the graph above is that error decreases with increasing flow rate. This insinuates that the IGT Equation could be more accurate for higher flowrates and pressures of natural gas.

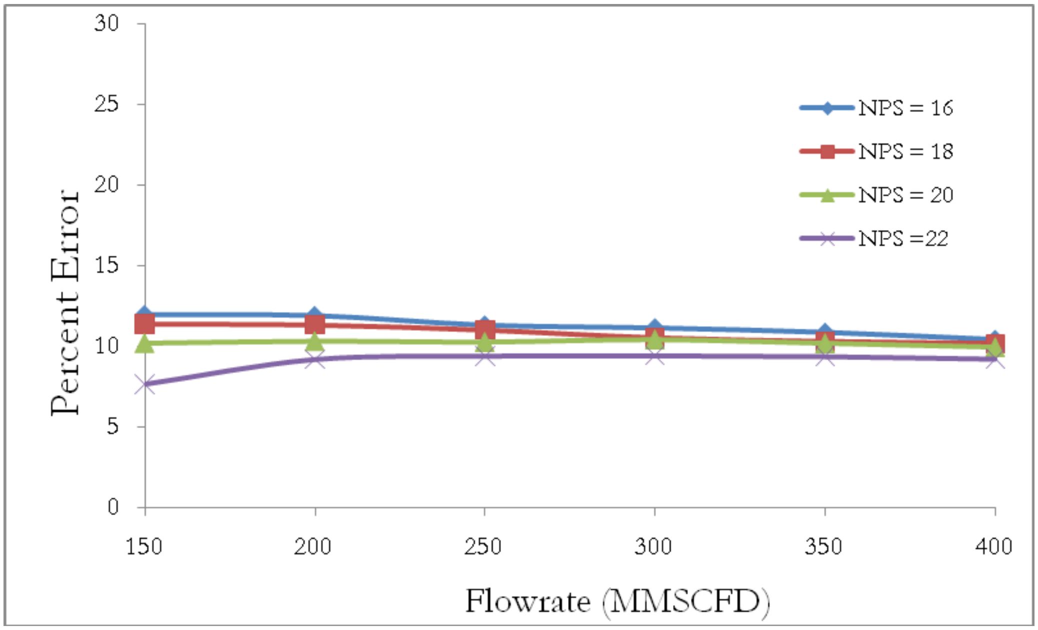
As done in previous equations, an elevation change of 0.15 km was applied to the Panhandle equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A23: Pressure Drop Comparison with Elevation -- IGT & Pro/II



The values obtained using the IGT Equation, with a 0.15 km elevation change, were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A24: Error Anaylsis with Elevation –IGT & Pro/II



The error remains significant when the elevation change is affecting the system. Conversely, an error trend that is noticeable in this graph is that the IGT equation produced more accurate results for larger diameter pipe. This trend only appears for the IGT Equation with an elevation change.

Spitzglass Equation

There are two forms of the Spitzglass Equation: one for low pressures (less than or equal to 1 pisg) and one for pressures higher than 1 psig. The low pressure equation is the following[2]:

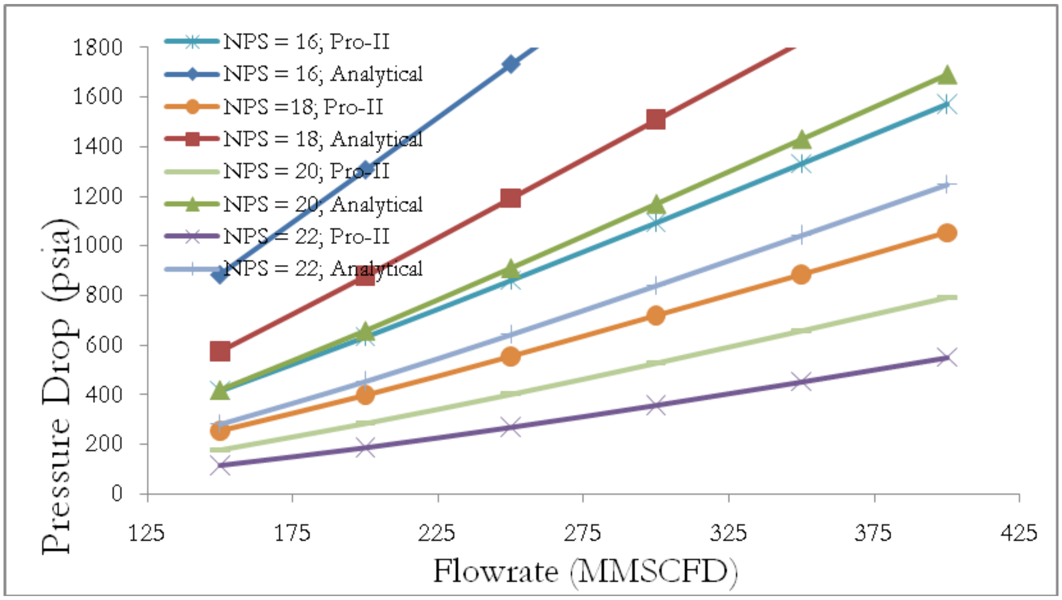
$$Q = 3.839 \times 10^{3} E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1 - P_2}{G T_f L_e Z\left(1 + \frac{3.6}{D} + 0.03D\right)}\right)^{0.5} D^{2.5} \quad \text{(USCS units)} \tag{A17}$$

The higher pressure equation is similar with slightly different constants[2]:

$$Q = 5.69 \times 10^{-2} E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1 - P_2}{G T_f L_e Z\left(1 + \frac{91.44}{D} + 0.0012D\right)}\right)^{0.5} D^{2.5} \quad \text{(SI units)} \tag{A18}$$

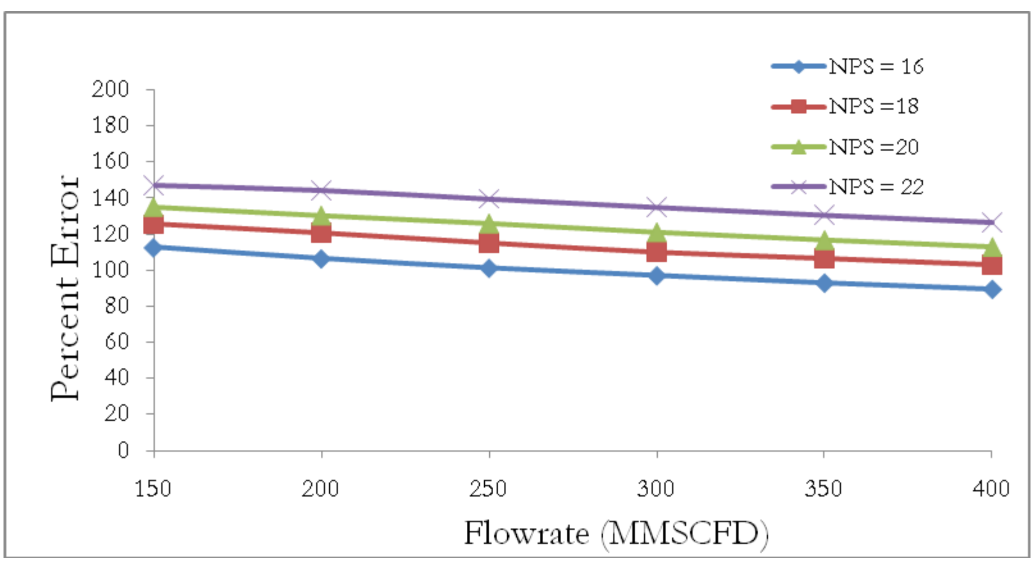
The Spitzglass Equation was implemented to determine a pressure drop in a pipe segment under various flowrates in a similar fashion as the other equations.

Figure A25: Pressure Drop Comparison – Spitzglass & Pro/II



The values obtained using the Spitzglass Equation were compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

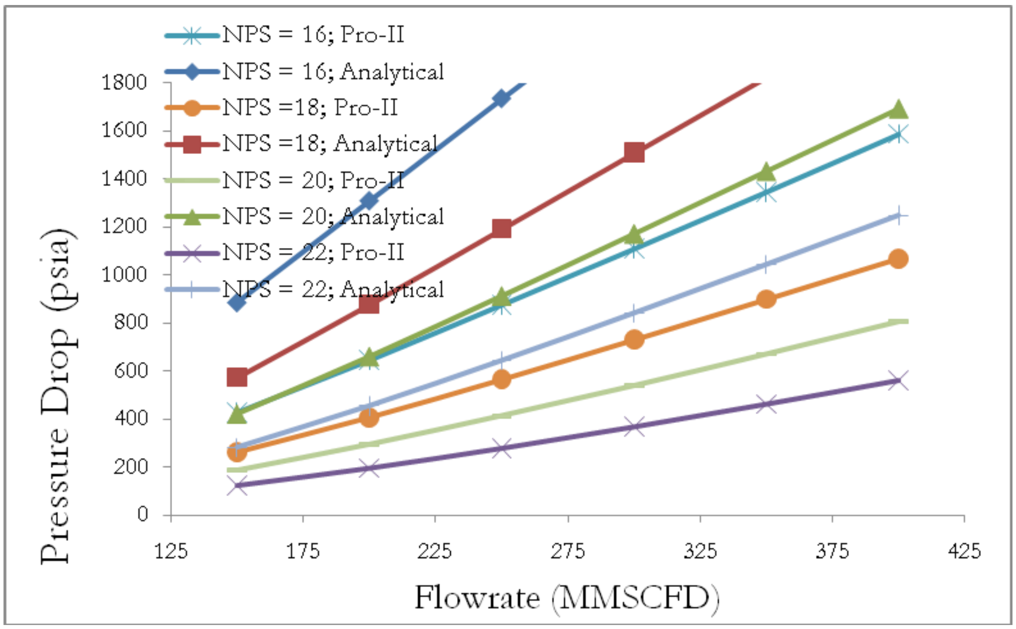
Figure A26: Error Anaylsis -- Spitzglass & Pro/II



The Spitzglass equation had an extremely large amount of error ranging from about 90-150%. There is however some noticeable trends that can be construed from this graph. It shows that the amount of error decrease in flowrates increase, and when pipe diameters increase. This could mean that the equation might eventually be an accurate equation for high flow rates/high pressure pipelines.

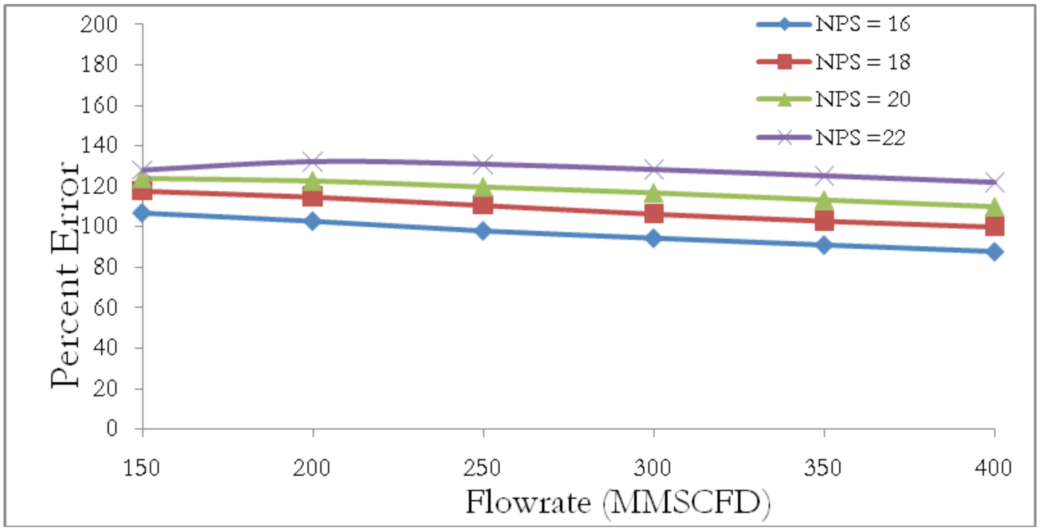
For consistency, an elevation change of 0.15 km was applied to the equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A27: Pressure Drop Comparison with Elevation -- Spitzglass & Pro/II



The values obtained using the Spitzglass Equation, with a 0.15 km elevation change, were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A28: Error Anaylsis with Elevation – Spitzglass & Pro/II



The error was less than the error created when no elevation was employed but was still extremely high ranging from 90-130%. Also, the elevation change produced the same trends mentioned in the graph with no elevation factor.

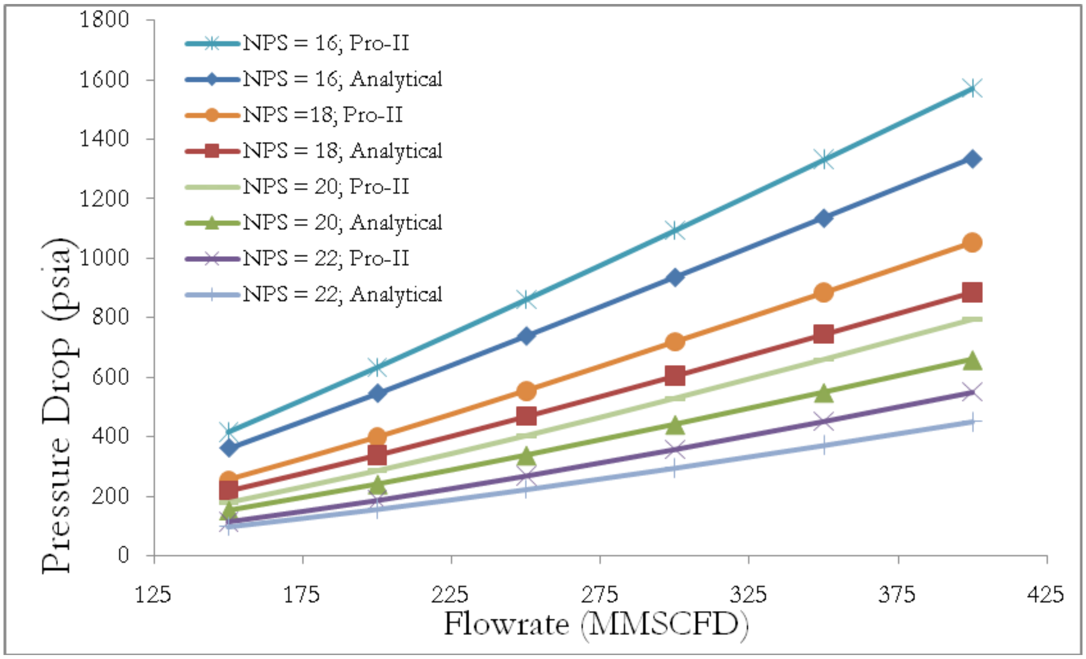
Mueller Equation

The Mueller Equation is another variation of the General Flow Equation that has the following form[2]:

$$Q = 85.7368E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^s P_2^2}{G^{0.7391}T_f L_e \mu^{0.2609}}\right)^{0.575} D^{2.725} \quad \text{(USCS units)} \tag{A19}$$

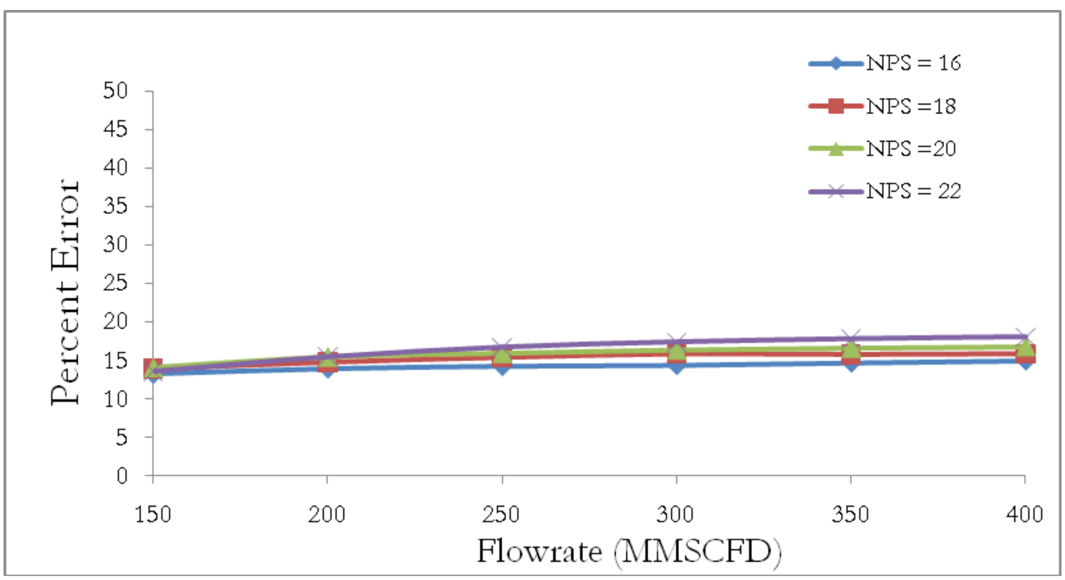
The Mueller Equation was implemented to determine a pressure drop in a pipe segment under various flowrates in a similar fashion as the other equations.

Figure A29: Pressure Drop Comparison – Mueller & Pro/II



The values obtained using the Mueller Equation were also compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

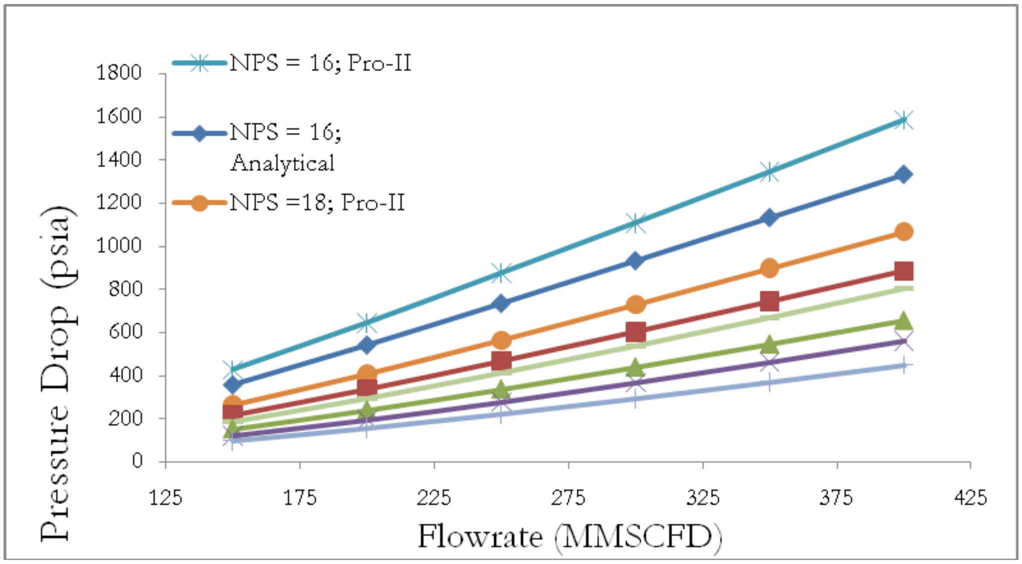
Figure A30: Error Anaylsis -- Mueller & Pro/II



The Mueller Equation had a large amount of error (13-18%) under the conditions analyzed. A couple of trends that are shown in the graph above is that the Mueller Equation is more accurate for smaller diameter pipe and for lower flow rates. This means that the equation could be accurate for systems with lower flow rates.

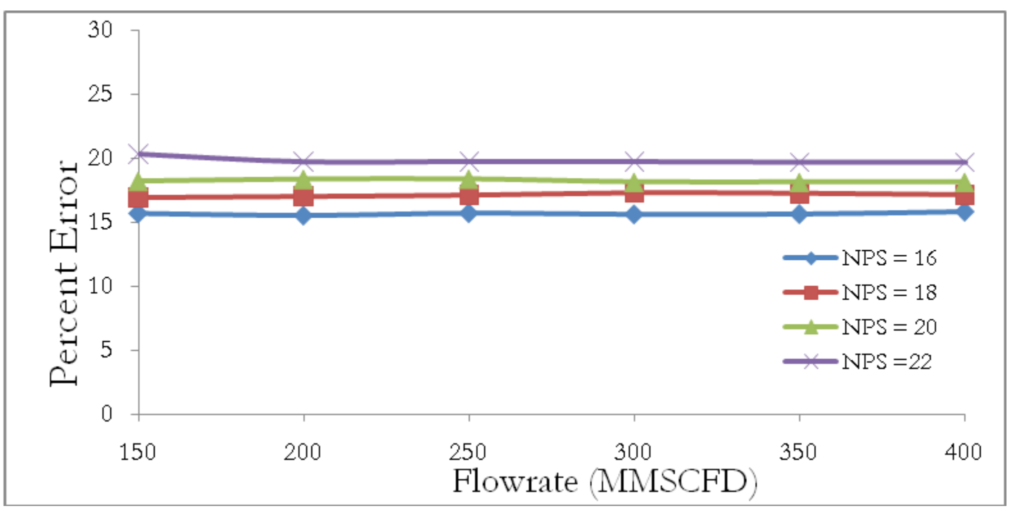
The values obtained using the Mueller Equation with a 0.15 km elevation change were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A31: Pressure Drop Comparison with Elevation -- Mueller & Pro/II



The values obtained using the Mueller Equation with a 0.15 km elevation change were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure A32: Error Anaylsis with Elevation – Mueller & Pro/II



The elevation change significantly affected the error produced with the Mueller Equation. Now it is shown to have an error range of around 15-20%. This proves that the Mueller Equation becomes less accurate for systems with large elevation changes.

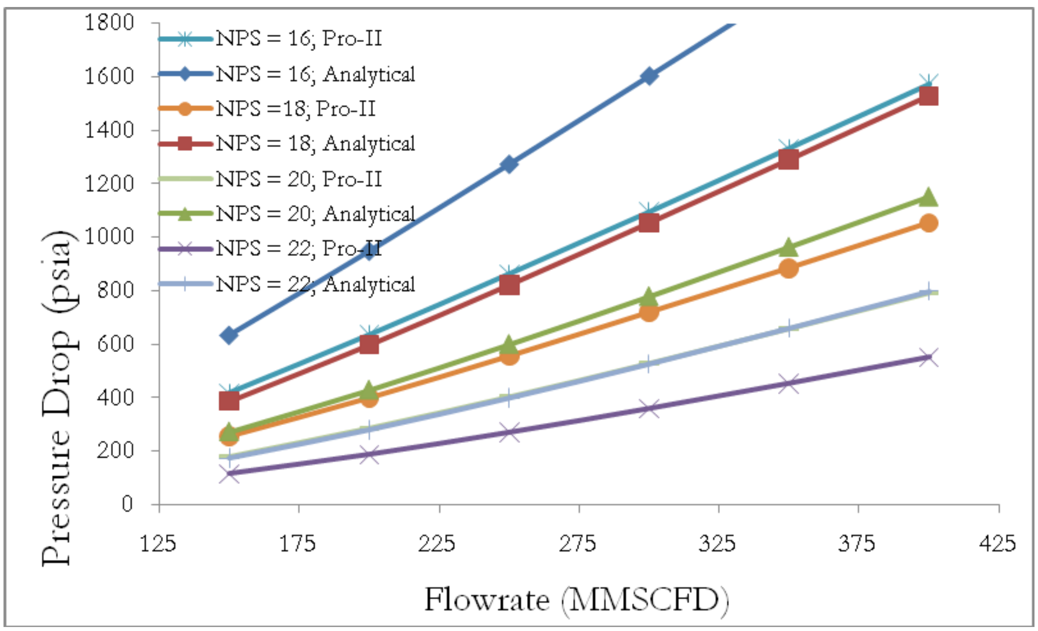
Fritzsche Equation

The last formula that is a variation of the General Flow Equation that was examined was the Fritzsche Equation. It has the following form[2]:

$$Q = 410.1688E\left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - P_2^2}{G^{0.8587}T_f L_e}\right)^{0.538} D^{2.69} \quad \text{(USCS units)} \tag{A20}$$

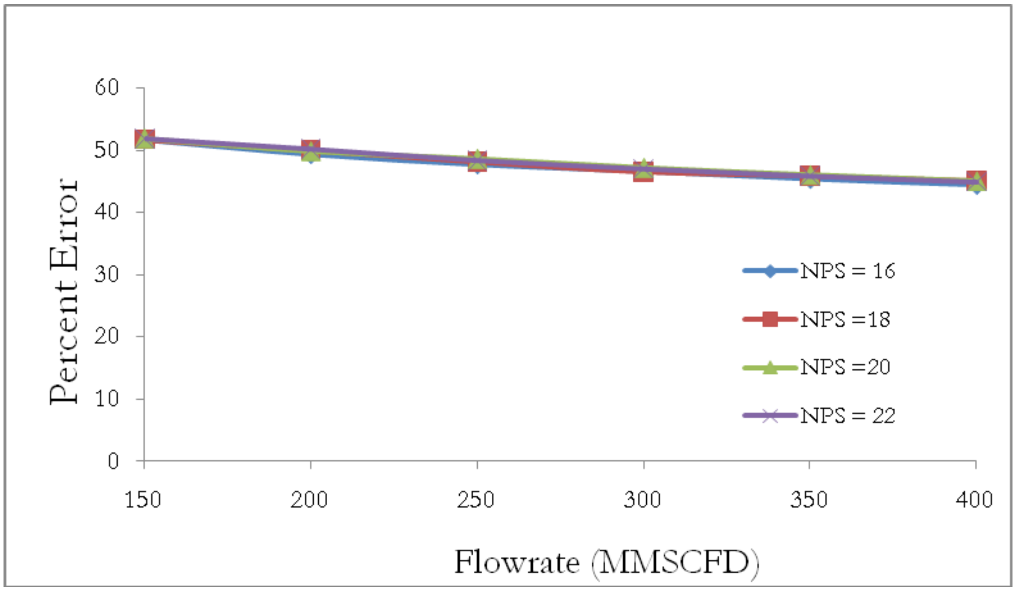
This equation was applied to similar conditions as the other equations were to produce the following graph.

Figure A33: Pressure Drop Comparison – Fritzsche & Pro/II



The values obtained using the Fritzsche Equation were also compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

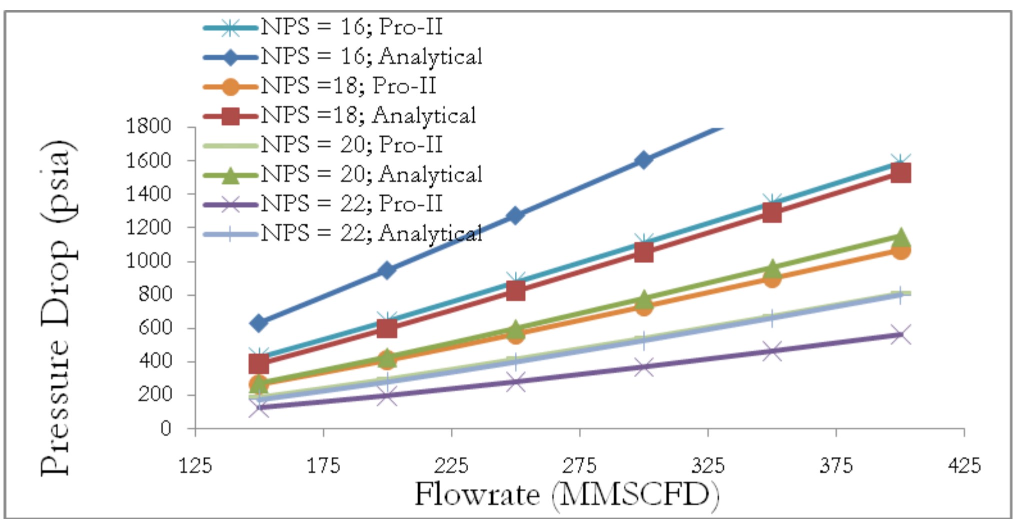
Figure A34: Error Anaylsis -- Fritzsche & Pro/II



The Fritzsche Equation was another equation that produced a large amount of error for the set conditions. This error ranged from about 45-50%. A noticeable trend in this data shows that the error decreases with an increasing flow rate. This suggests that the Fritzche Equation will become more accurate for systems with large flowrates and high pressures.

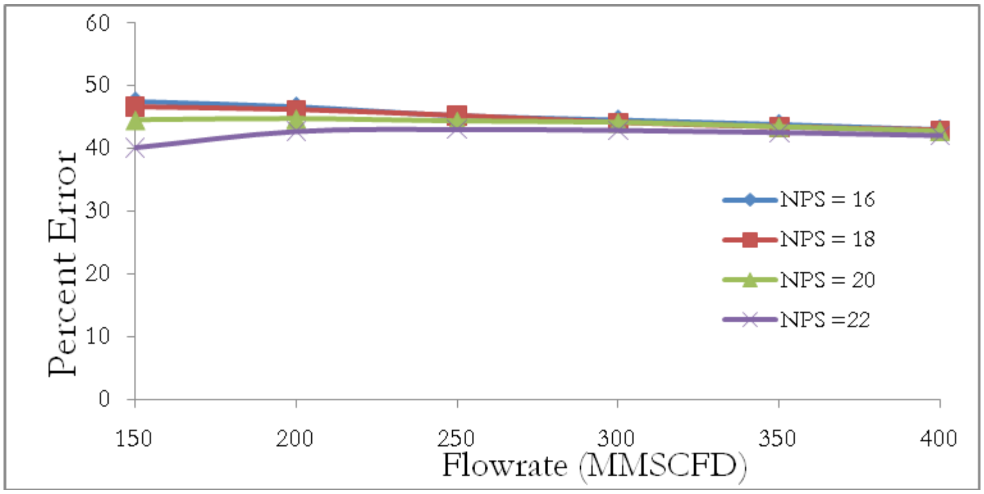
For consistency, an elevation change of 0.15 km was applied to the equation in order to determine how much elevation can affect the accuracy of the equation.

Figure A35: Pressure Drop Comparison with Elevation -- Fritzsche & Pro/II



The values obtained using the Fritzsche Equation with a 0.15 km elevation change were again compared to Pro/II in order to determine how accurate the expression can compute the pressure at various flow rates.

Figure 36: Error Anaylsis with Elevation – Fritzsche & Pro/II



The amount of error did not change that much for the Fritzsche Equation applied to a system with an elevation change. It has slightly less amount of error with a range from 40-48%. Also, the larger diameter pipe gave more accurate pressure drop values. This only occurred when the elevation was applied to the equation.

Hydraulic Equations Analysis Conclusion

In summation, it is clear that pipeline design is at the very least a tricky and somewhat frustrating task to undertake. It would appear that industry combats this by settling on a method and "biting the bullet" on the error in cost analysis. Industry has come to accept an unnecessary margin of error that can theoretically be minimized if not eliminated altogether. Presented here is a small collection of the most popular correlations available to industry. If other equations not evaluated here have similar errors to these, then the industry is truly without a reliable tool to optimize an infinitely diverse and dynamic market.

Appendix B

J-Curve Analysis on Natural Gas Pipelines

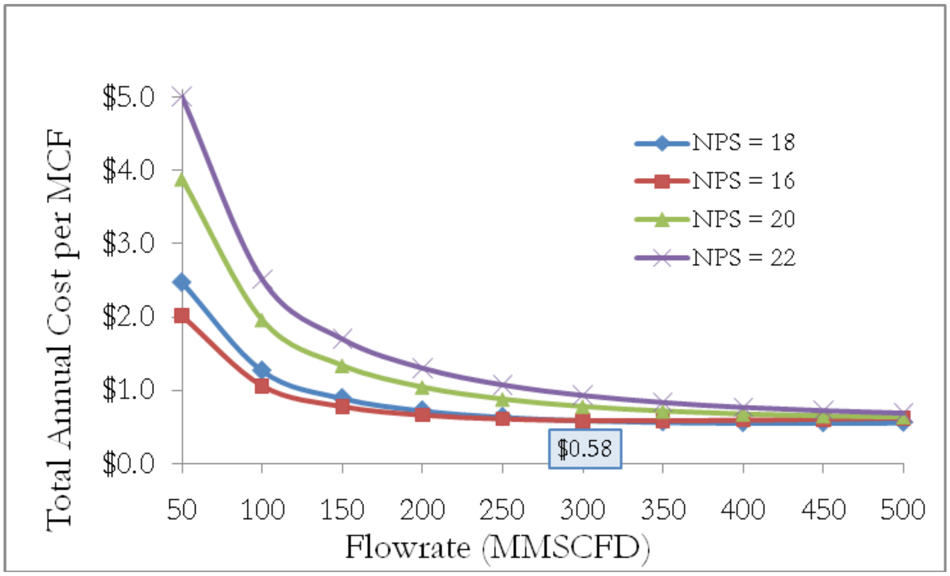
Introduction to J-Curves

According to natural gas pipeline literature, J-curves are a useful tool in the preliminary stages of pipeline design (Menon). J-curves, in relation to the Natural Gas Industry, provide an economic analysis for pipelines as a function of pipeline diameter and natural gas flow rate. While J-curves may be particle to optimize extremely simple pipeline systems (i.e. one or two pipes in series with one supplier and one consumer and minimal possibilities for compressor locations; the in depth analysis of J-curves in this report proves that this task becomes exponentially more difficult for complex, ramified pipeline networks. Also, by implementing J-curves for optimization an analyzer introduces several risky assumptions about the network. Thus, the purpose of this report is to introduce J-Curves, and apply these optimizing tools to simple pipeline networks. In doing so, the reader will obtain a better understanding of if and when J-curve optimization can be used in the natural gas industry.

Overview of J-Curve Analysis

A J-curve, for natural gas pipeline purposes, is most simplistically a graph illustrating the cost of a defined size pipeline at varying flow rates. Graphing multiple diameters on the same graph allows the analyzer to select the lowest cost at a given demand flow rate. These curves are named J-curves due to the fact that they visually appear similar to a section on the lower half of the letter "J". Figure B1 below is an example of a J-curve.

Figure B1: Example of a J-curve



In this analysis, several assumptions are made: there is no volume buildup within the pipe, the facilities are designed to meet the design volume, the time value of money is neglected, and inflation is ignored. Operating costs, maintenance costs, fuel costs, depreciation, taxes, and return on investment are all assumed constant. These assumptions are unacceptable if the pipeline designer wants to perform a thorough investigation of the economics of a pipeline.

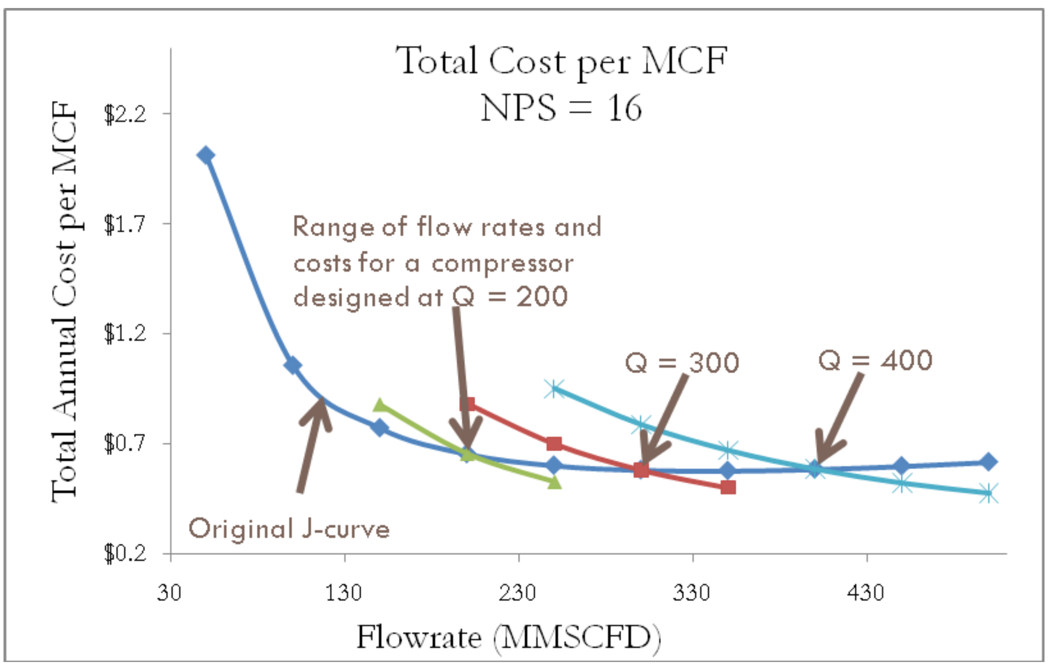
Furthermore, the J-curve procedure is quite simple. First, the pipe size, maximum operating pressure, pipeline length, and compression ratio are fixed. Calculations are then completed for varying flow rates. When the correct pipeline hydraulic simulation software is available, the results are obtained using simulation techniques rather than other means of calculation. At this point, an economical analysis is performed which yields the total annual cost for the varying flow rates. When this total annual cost is plotted against flow rate, the resulting graph is a J-curve. By repeating these steps, multiple J-curves are created. The lowest of these curves relative to one another near the flowrate range of interest is the economic optimum. Often, this procedure is repeated while varying operating pressure in order to obtain an optimum pressure at a desired flowrate and pipe size. Also, if there are multiple sections of a ramified pipeline, the order of pipeline segments affects the results for the overall optimized specifications. Thus, all possible orders in which the sections are analyzed must be produced to find the true optimal design in a complex system. Therefore, this procedure can become quite troublesome for a designing a complex pipeline network.

Spring 2008 Analysis

During the spring of 2008, Chase Waite and Kristy Booth along with graduate student Debora Campos de Faria took an in-depth look at J-curves and their shortcomings (Bagajewicz).

The study paid special attention to the J-curve method's lack of compensation for compressor efficiency and selection. On a J-curve, each point is considered to be a unique compressor; that is, the cost of the compressor is only for a compressor sized at that particular flow rate. As the demand flowrate varies, the cost of the compressor does NOT vary according to the J-curve. This is shown below in Figure B2.

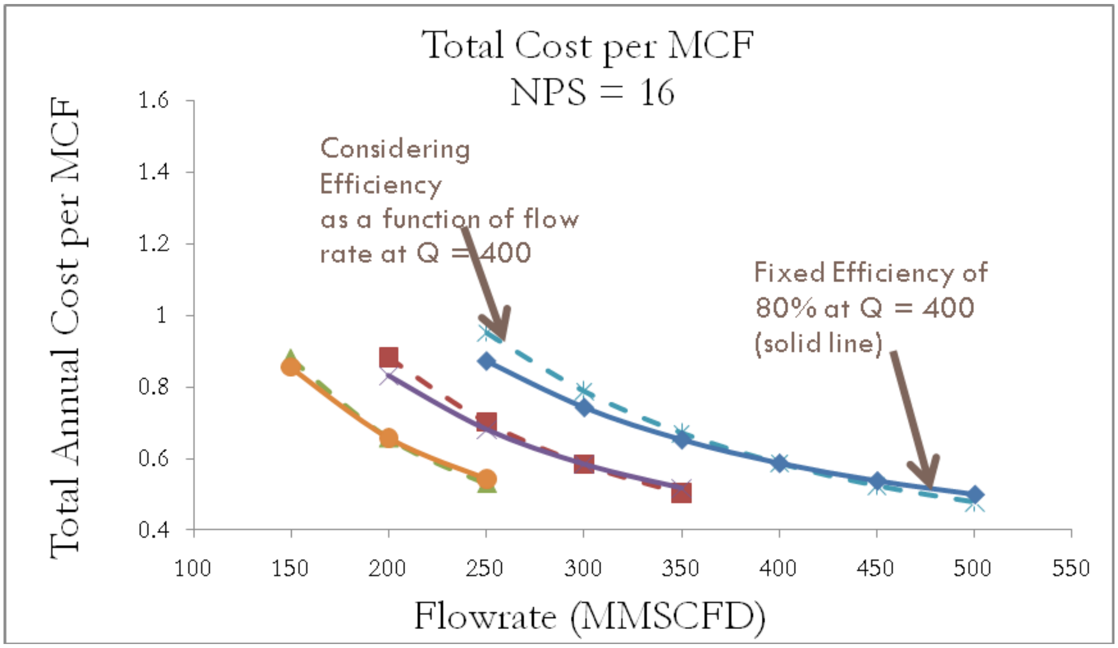
Figure B2: Effects of Flowrate Variations on Cost



It is observed that a decrease in flowrate increases the total annual cost while increasing the flowrate decreases the annual cost. On the other hand, operating the compressor over the design parameters can harm the equipment.

The study also looked at the effects of compressor efficiency on the reliability of J-curves. Variation in compressor efficiency can drastically affect the total annual cost. This is illustrated by Figure B3 below.

Figure B3: Effects of Compressor Efficiency on Cost



It is observed that a decrease in the design flowrate increases the cost of the project. This throws into question how a designer is to select the best parameters for the compressor.

There are many other disadvantages to the J-curve method. Most importantly, this analysis only works for very simple systems. This is highly unrealistic. It is assumed the complex systems are optimized with this system by evaluating each individual pipe segment separately. This is extremely time consuming, and not all that reliable in the end. Also, as the systems become more complex, the J-curve analysis becomes exponentially more time-consuming and inaccurate. It is possible that J-curves can cross. This leaves the analyzer in a predicament.

Many other disadvantages and shortcomings exist. Just to name a few: the method does not account for volume variations, load factors, the time value of money, etc.

Appendix C

Linear Model Expansion and Revision

During a meeting with Willbros, Inc., on Friday, February 20th, 2008, Willbros engineers made various suggestions regarding how to make the mathematical model more industry-friendly and industry-useful. One of their immediate concerns was the lack of diameter options. They also mentioned that pipe coating costs, material transportation costs, and an option for quadruple random length joints costs were not included in the model. Other expansions have also been analyzed such as installation cost, pipe maintenance cost, and compressor maintenance cost.

Diameter Expansion

The previous capstone group working on this project evaluated four diameters: 16, 18, 20, and 24 in. Willbros claims that these are actually rather small diameters compared to those used in most of their industrial applications. So an expansion of this range is necessary. Even pipe diameters from 2-42 in. are evaluated.

Willbros next mentioned that pricing and sizing is usually only available for industry-standard pipe sizes and schedules, so evaluating each and every even pipe size within the given range requires generating some wall thickness and cost data. Willbros has confirmed that pipe can be made to meet buyer specifications at only the price of the steel (no fee) given that the pipe is greater than one mile long.

Also, new values of A and B for the mathematical model must be generated because these values are dependent upon diameter.

Analysis

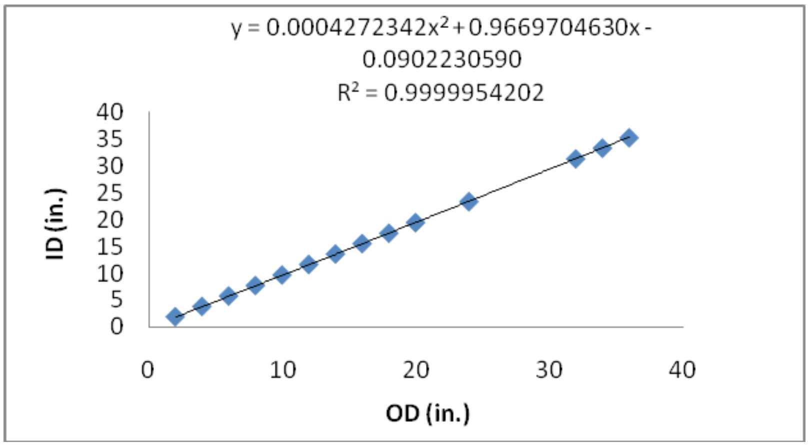
The first task is to collect diameter data for the pipe sizes at hand. For most, the sizes are readily available. Only schedule 40 pipe is evaluated. For non-standard pipe sizes, a 2nd order polynomial regression analysis is performed to predict the wall thicknesses not published. Table C1 gives industry standard pipe sizes.

Table C1: Standard Schedule 40 Pipe Sizes

OD (in.) ID (in.)	
2 1.846	
4 3.763	
6 5.72	
8 7.678	
10 9.635	
12 11.594	
14 13.563	
16 15.5	
18 17.438	
20 19.407	
24 23.313	
32 31.312	

These values were plotted in order predict the pipe parameters not published. A 2nd order polynomial regression of standard pipe sizes gives a sufficient R^2 value. Figure C1 below shows the regression.

Figure C1: Regression of Industry-Standard Schedule 40 Pipe Sizes



These equations are then used to predict pipe size parameters for non-standard pipe sizes. Table C2 below shows the results. Values with asterisks are those predicted by the regression.

Table C2: Schedule 40 Standard Pipe Sizes

OD (in.) ID (in.)	
2 1.846	
4 3.763	
6 5.720	
8 7.678	
10 9.635	
12 11.594	
14 13.563	
16 15.500	
18 17.438	
20 19.407	
***22 21.390**	
24 23.313	

Now that the pipe parameters are known, prices can be determined. The pipe price is provided by Omega Steel through a spreadsheet provided to last year's capstone group. The pipe price for December 2008 is determined to be 513 $/ton for hot rolled steel coil (SteelontheNet).

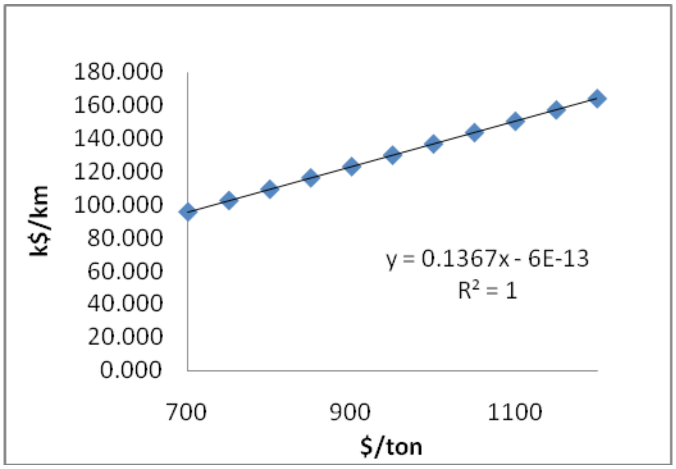
The spreadsheet only gives prices for 700-1200 $/ton, so an extrapolation technique is used to find the price at 513 $/ton. Table C3 below is an example for the 16 in. OD pipe.

Table C3: Pricing for 16 in. OD Pipe

OD (in.) **$/ton** **$/ft** **k$/km**			
16 700 28.997 95.689			
750 31.068 102.524			
ID (IN.) 800 33.139 109.359			
15.500 850 35.210 116.194			
900 37.281 123.029			
W.T. (IN.) 950 39.353 129.863			
0.500			

A regression analysis is then performed ($/ton vs. k$/km). Figure C2 below shows the regression.

Figure C2: Price Regression for 16 in. OD Pipe



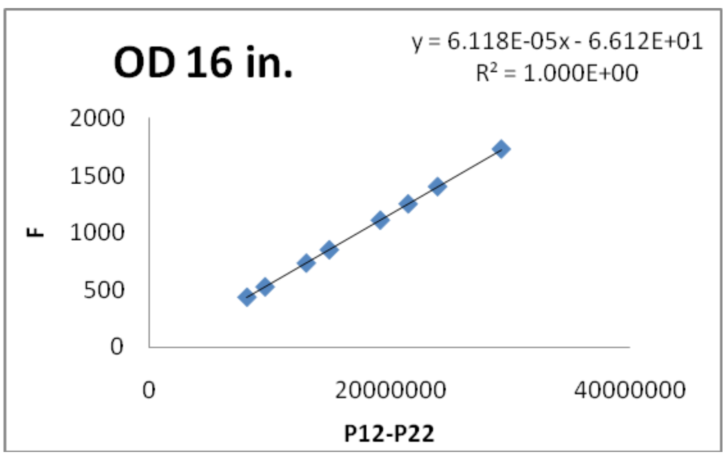
The regression equation is then used to find the k$/km at the selected steel price of 513 $/ton. The results for all pipe sizes are shown below in Table C4.

Table C4: Price for Schedule 40 Pipe

OD (in.) k$/km	
2 3.0780	
4 9.1314	
6 16.0569	
8 24.2136	
10 34.3197	
12 42.5790	
14 53.6085	
16 70.1271	
18 88.6977	
20 104.1390	
22 118.0413	
24 144.9225	

Finally, the new values for A and B are generated. This is done using a linear regression in Excel. flowrate is changed while length and elevation change are held constant. A graph of $\frac{Q*L^2}{D^5}$ vs. $P_1^{\,2} - P_2^{\,2}$ gives A as the slope and $\frac{B}{-\Delta z}$ as the intercept. The graph for OD 16 in. is shown below in Figure C3.

Figure C3: Regression for OD 16 in.



The inputs and the resulting A and B values are shown below in Table C5.

Table C5: 16 in. OD A & B Values

ID (in.)	Q (Mcmd)	L(km)	F	P1 (kPa)	P2 (kPa)	P12-P22	z	slope	intercept	A	B
15.500	1415.8423	193.1213	432.7145	5589.594	6272.892	8105613	0.15	6.118E-05	-6.612E+01	6.118E-05	4.408E+02
15.500	1557.4266		523.5845	5590.545	6393.081	9617291					
15.500	1840.595		731.2875	5589.78	6655.695	13052644					
15.50	1982.17		848.12	5591.11	6799.66	149748					

0	93		04	7	5	49					
15.500	2265.3477		1107.749	5589.98	7104.344	19223833					
15.500	2406.932		1250.545	5594.034	7269.239	21548624					
15.500	2548.5162		1401.995	5601.928	7442.249	24005474					
15.500	2831.6847		1730.858	5615.476	7800.377	29312302					

This is then repeated for each diameter. The results are shown below in Table C6.

Table C6: A & B Values for All Pipe Diameters

OD (in.)	A	B
2	4.003E-05	7.633E+02
4	4.125E-05	2.235E+02
6	4.984E-05	3.859E+02
8	5.096E-05	2.831E+02
10	5.700E-05	5.757E+02
12	5.744E-05	3.795E+02
14	6.086E-05	6.313E+02
16	6.118E-05	4.408E+02
18	6.350E-05	4.817E+02
20	6.383E-05	4.003E+02
22	6.678E-05	5.833E+02
24	6.693E-05	4.737E+02
26	6.721E-05	4.151E+02
28	6.758E-05	3.865E+02

30	6.801E-05	3.714E+02
32	6.849E-05	3.639E+02
34	6.897E-05	3.597E+02
36	6.943E-05	3.582E+02
38	6.990E-05	3.575E+02
40	7.034E-05	3.574E+02
42	7.073E-05	3.575E+02

Implementation

Now that the prices have been determined, these values can be implemented into the mathematical programming model. The first step is to expand the diameter "set". The change is shown below.

Before

```
d  pipe diameter options /16,18,20,22/;
```

After

```
d  pipe diameter options /2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42/;
```

The next step is to update the pipeline inner diameter and pipeline cost "parameters". This is shown below.

Before

```
ID(d) Pipeline options inner diameter (in)
     / 24   22.62
       28   26.75
       32   30.62
       36   34.5/
```

```
CP(d) Pipeline options cost (k$ per Km)
     / 24   742.993
       28   850.005
       32   1046.679
       36   1300.620/
```

After

```
ID(d) Pipeline options inner diameter SCH40(in)
    /
        2     1.846
        4     3.763
        6     5.720
        8     7.678
        10    9.635
        12    11.594
        14    13.563
        16    15.500
        18    17.438
        20    19.407
        22    21.390
        24    23.313
        26    25.340
        28    27.320
        30    29.303
        32    31.312
        34    33.312
        36    35.250
        38    37.272
        40    39.272
        42    41.276
                   /
```

```
CP(d) Pipeline steel cost SCH40(k$ per Km)
   /
         2     3.0780
         4     9.1314
         6     16.0569
         8     24.2136
         10    34.3197
         12    42.4790
         14    53.6085
         16    70.1271
         18    88.6977
         20    104.1390
         22    118.0413
         24    144.9225
         26    151.3350
         28    168.1101
         30    184.8339
         32    194.9400
         34    207.4059
         36    239.2119
         38    245.5218
         40    258.7059
         42    270.4023
                     /
```

Lastly, the new A and B values are inserted. This change is shown below.

Before

After

```
****Setting A and B values
AC('16',c,ca)=6.118E-5*XPCC(c,ca);
AC('18',c,ca)=6.350E-5*XPCC(c,ca);
AC('20',c,ca)=6.383E-5*XPCC(c,ca);
AC('24',c,ca)=6.693E-5*XPCC(c,ca);
BC('16',c,ca)=4.408E+2*XPCC(c,ca);
BC('18',c,ca)=4.817E+2*XPCC(c,ca);
BC('20',c,ca)=4.003E+2*XPCC(c,ca);
BC('24',c,ca)=4.737E+2*XPCC(c,ca);
AS('16',s,c)=6.118E-5*XPSC(s,c);
AS('18',s,c)=6.350E-5*XPSC(s,c);
AS('20',s,c)=6.383E-5*XPSC(s,c);
AS('24',s,c)=6.693E-5*XPSC(s,c);
BS('16',s,c)=4.408E+2*XPSC(s,c);
BS('18',s,c)=4.817E+2*XPSC(s,c);
BS('20',s,c)=4.003E+2*XPSC(s,c);
BS('24',s,c)=4.737E+2*XPSC(s,c);
```

```
****Setting A and B values
AC('2',c,ca)=4.003E-5*XPCC(c,ca);
AC('4',c,ca)=4.125E-5*XPCC(c,ca);
AC('6',c,ca)=4.984E-5*XPCC(c,ca);
AC('8',c,ca)=5.096E-5*XPCC(c,ca);
AC('10',c,ca)=5.700E-5*XPCC(c,ca);
AC('12',c,ca)=5.744E-5*XPCC(c,ca);
AC('14',c,ca)=6.086E-5*XPCC(c,ca);
AC('16',c,ca)=6.118E-5*XPCC(c,ca);
AC('18',c,ca)=6.350E-5*XPCC(c,ca);
AC('20',c,ca)=6.383E-5*XPCC(c,ca);
AC('22',c,ca)=6.678E-5*XPCC(c,ca);
AC('24',c,ca)=6.693E-5*XPCC(c,ca);
AC('26',c,ca)=6.721E-5*XPCC(c,ca);
AC('28',c,ca)=6.758E-5*XPCC(c,ca);
AC('30',c,ca)=6.801E-5*XPCC(c,ca);
AC('32',c,ca)=6.849E-5*XPCC(c,ca);
AC('34',c,ca)=6.897E-5*XPCC(c,ca);
AC('36',c,ca)=6.943E-5*XPCC(c,ca);
AC('38',c,ca)=6.990E-5*XPCC(c,ca);
AC('40',c,ca)=7.034E-5*XPCC(c,ca);
AC('42',c,ca)=7.073E-5*XPCC(c,ca);
BC('2',c,ca)=7.633E+2*XPCC(c,ca);
BC('4',c,ca)=2.235E+2*XPCC(c,ca);
BC('6',c,ca)=3.859E+2*XPCC(c,ca);
BC('8',c,ca)=2.831E+2*XPCC(c,ca);
BC('10',c,ca)=5.757E+2*XPCC(c,ca);
BC('12',c,ca)=3.795E+2*XPCC(c,ca);
BC('14',c,ca)=6.313E+2*XPCC(c,ca);
BC('16',c,ca)=4.408E+2*XPCC(c,ca);
BC('18',c,ca)=4.817E+2*XPCC(c,ca);
BC('20',c,ca)=4.003E+2*XPCC(c,ca);
BC('22',c,ca)=5.833E+2*XPCC(c,ca);
BC('24',c,ca)=4.737E+2*XPCC(c,ca);
BC('26',c,ca)=4.151E+2*XPCC(c,ca);
BC('28',c,ca)=3.865E+2*XPCC(c,ca);
BC('30',c,ca)=3.714E+2*XPCC(c,ca);
BC('32',c,ca)=3.639E+2*XPCC(c,ca);
BC('34',c,ca)=3.597E+2*XPCC(c,ca);
BC('36',c,ca)=3.582E+2*XPCC(c,ca);
BC('38',c,ca)=3.575E+2*XPCC(c,ca);
BC('40',c,ca)=3.574E+2*XPCC(c,ca);
BC('42',c,ca)=3.575E+2*XPCC(c,ca);
```

```
AS('2',s,c)=4.003E-5*XPSC(s,c);
AS('4',s,c)=4.125E-5*XPSC(s,c);
AS('6',s,c)=4.984E-5*XPSC(s,c);
AS('8',s,c)=5.096E-5*XPSC(s,c);
AS('10',s,c)=5.700E-5*XPSC(s,c);
AS('12',s,c)=5.744E-5*XPSC(s,c);
AS('14',s,c)=6.086E-5*XPSC(s,c);
AS('16',s,c)=6.118E-5*XPSC(s,c);
AS('18',s,c)=6.350E-5*XPSC(s,c);
AS('20',s,c)=6.383E-5*XPSC(s,c);
AS('22',s,c)=6.678E-5*XPSC(s,c);
AS('24',s,c)=6.693E-5*XPSC(s,c);
AS('26',s,c)=6.721E-5*XPSC(s,c);
AS('28',s,c)=6.758E-5*XPSC(s,c);
AS('30',s,c)=6.801E-5*XPSC(s,c);
AS('32',s,c)=6.849E-5*XPSC(s,c);
AS('34',s,c)=6.897E-5*XPSC(s,c);
AS('36',s,c)=6.943E-5*XPSC(s,c);
AS('38',s,c)=6.990E-5*XPSC(s,c);
AS('40',s,c)=7.034E-5*XPSC(s,c);
AS('42',s,c)=7.073E-5*XPSC(s,c);
BS('2',s,c)=7.633E+2*XPSC(s,c);
BS('4',s,c)=2.235E+2*XPSC(s,c);
BS('6',s,c)=3.859E+2*XPSC(s,c);
BS('8',s,c)=2.831E+2*XPSC(s,c);
BS('10',s,c)=5.757E+2*XPSC(s,c);
BS('12',s,c)=3.795E+2*XPSC(s,c);
BS('14',s,c)=6.313E+2*XPSC(s,c);
BS('16',s,c)=4.408E+2*XPSC(s,c);
BS('18',s,c)=4.817E+2*XPSC(s,c);
BS('20',s,c)=4.003E+2*XPSC(s,c);
BS('22',s,c)=5.833E+2*XPSC(s,c);
BS('24',s,c)=4.737E+2*XPSC(s,c);
BS('26',s,c)=4.151E+2*XPSC(s,c);
BS('28',s,c)=3.865E+2*XPSC(s,c);
BS('30',s,c)=3.714E+2*XPSC(s,c);
BS('32',s,c)=3.639E+2*XPSC(s,c);
BS('34',s,c)=3.597E+2*XPSC(s,c);
BS('36',s,c)=3.582E+2*XPSC(s,c);
BS('38',s,c)=3.575E+2*XPSC(s,c);
BS('40',s,c)=3.574E+2*XPSC(s,c);
BS('42',s,c)=3.575E+2*XPSC(s,c);
```

Cost Expansion

Problem

Previous economic analysis of the mathematical program has shown that many aspects of pipeline design economics are not contained within the model. Willbros has confirmed this and expanded the list of missing parameters. Previous analysis has shown that the model does not contain corrections for installation cost, pipe maintenance cost, and compressor maintenance cost while Willbros has suggested the need for corrections for coating cost, transportation cost, and quadruple random length joint cost.

Analysis

Installation Cost

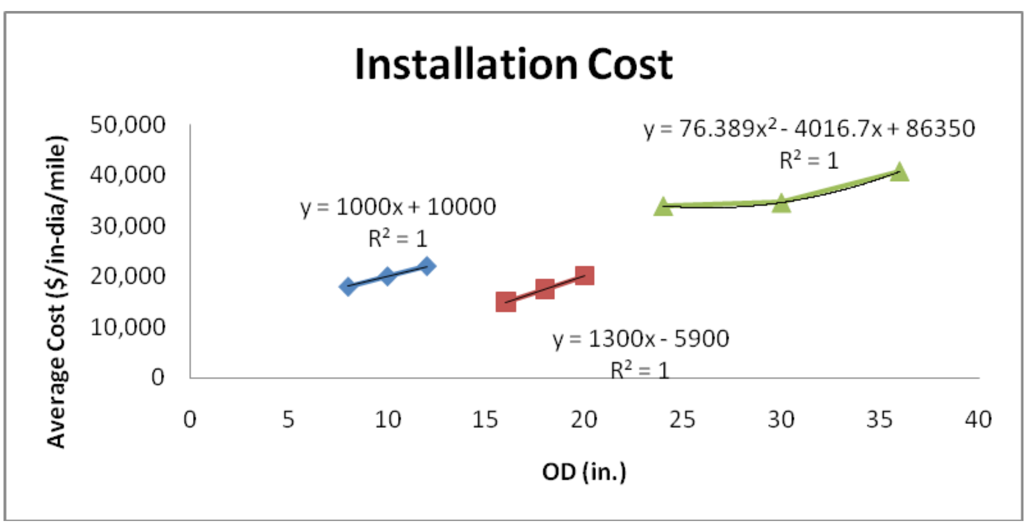
Pipeline installation cost data is given in Menon for various standard pipe sizes. The data provided is shown below in Table C7.

Table C7: Installation Costs

Pipe Diameter (in.)	Average Cost ($/in-dia/mile)
8	18,000
10	20,000
12	22,000
16	14,900
18	17,500
20	20,100
24	33,950
30	34,600
36	40,750

A regression analysis is used in order to obtain installation costs for non-standard pipe sizes. The data does not appear to follow a logical trend, so the data is separated into sections which seem to follow a trend. This variation is assumed to be evidence of the effects of increased equipment and power needs at various ranges of diameter. The regressions are shown below in Figure C4.

Figure C4: Installation Cost



Using the various regression equations, installation costs for non-standard pipe sizes can be calculated. These values are shown below in Table C8.

Table C8: Installation Cost for All Pipe Sizes

Pipe Diameter (in.)	Average Cost (k$/km)
2	14.913
4	34.797
6	59.652
8	89.477
10	124.274
12	164.042
14	208.781
16	148.135
18	195.732
20	249.791
22	310.313
24	506.293
26	542.099
28	587.568
30	644.983
32	716.574
34	804.668
36	911.552
38	1039.431
40	1190.657
42	1367.482

Pipeline and Compressor Maintenance Cost

The pipeline maintenance cost is inserted into the model as a percentage of the total annual cost. This is assumed because of its complexity and unpredictability. It is difficult or perhaps impossible to express the pipeline maintenance cost as a function of design parameters. Various sources have used a percentage of 0.5% TAC to compensate for pipeline maintenance (Parkinson). The cost of maintaining compressor stations is given as 16 $/hp (Mohitpour et.al.).

Coating Cost

In order to compensate for the cost of coating a pipeline, Willbros has provided a value of 6.5 $/ft. for a 36 in. OD pipe. This is the price for coating the entire pipe with Fusion Body Epoxy (FBE).

In order to use this value for other diameters, the ratio of diameters is used. This equation is shown below.

$$Cost(OD) = RefCost(OD) * \frac{OD}{RefOD} \tag{C1}$$

Thus, the coating cost for both standard and non-standard pipe sizes is calculated. The resulting values are shown below in Table C10.

Table C9: Coating Cost

OD(in.)	k$/km
2	1.184748
4	2.369495
6	3.554243
8	4.738991
10	5.923739
12	7.108486
14	8.293234
16	9.477982
18	10.66273
20	11.84748
22	13.03223
24	14.21697
26	15.40172
28	16.58647
30	17.77122
32	18.95596
34	20.14071
36	21.32546
38	22.51021
40	23.69495
42	24.8797

Transportation Cost

Transportation cost is found in much the same manner. A value of 6 $/ft for a 36 in. OD pipe has been provided by Willbros. The same diameter ratio analysis is used to predict transportation cost for the other pipe sizes. The results are shown below in Table C10.

Table C10: Transportation Cost for All Pipe Sizes

OD(in.)	k$/km
2	1.093613
4	2.187227
6	3.28084
8	4.374453
10	5.468067
12	6.56168
14	7.655293
16	8.748906
18	9.84252
20	10.93613
22	12.02975
24	13.12336
26	14.21697
28	15.31059
30	16.4042
32	17.49781
34	18.59143
36	19.68504
38	20.77865
40	21.87227
42	22.96588

Quadruple Random Length Joint Cost

Adding quadruple random length joints is a labor-minimizing method used in pipeline construction. The method involves reducing welding in the field by doing more welding in the mill. There is a cost for this service although it is economically sound because of reduced field labor. The value has been provided by Willbros as approximately 1800 $/km.

Implementation

Installation Cost

Installation cost is added to the mathematical linear model by introducing a new "parameter". This is shown below.

```
PIC(d) Pipeline installation cost (k$ per km)
/
    2     14.913
    4     34.797
    6     59.652
    8     89.477
    10    124.274
    12    164.042
    14    208.781
    16    148.135
    18    195.732
    20    249.791
    22    310.313
    24    506.293
    26    542.099
    28    587.568
    30    644.983
    32    716.574
    34    804.668
    36    911.552
    38    1039.431
    40    1190.657
    42    1367.4820
                    /
```

This new parameter must then be implemented into the economics of the program. This is shown below.

```
Pipecost(t)    ..Pipe(t)=E=sum((c,ca),LCC(c,ca)*sum(d,YDCC(c,ca,d,t)*(CP(d)+qr1+CC(d)+CT(d)+PIC(d))))
                        + sum((s,c),LSC(s,c)*sum(d,YDSC(s,c,d,t)*(CP(d)+qr1+CC(d)+CT(d)+PIC(d))));
```

As can be seen, the pipe installation cost is simply added to the cost of the pipe by multiplying the cost in k$/km by the length in km of a given section of pipe.

Pipeline Maintenance Cost

Because the pipeline maintenance cost is a set value, it has been implemented as a "scalar" such that it is easily changed to suit the user's purpose. This is shown below.

```
     pm         Pipeline maintenance facter (%TAC)                    / 0.005 /
* Set mc and pm to 0 if maintenance is included in the operating cost
```

It is not uncommon for pipeline maintenance cost to be included in the operating cost. Thus, a comment has been added telling the user to set the value to zero if the cost of maintenance is included in the operating cost. Next, this value must be added into the economics. A new equation is constructed which gives the pipeline maintenance cost as a fraction of the previous time period's total annual cost.

```
pmaintcost(t)            ..pmcost(t)=E=pm*(sum(tt$(ord(tt)<=ord(t)),(TAC(tt)*((1+i*eval)**(ord(t)-ord(tt))))));
```

Compressor Maintenance Cost

Compressor maintenance cost is implemented in much the same manner as pipeline maintenance cost. It is implemented as a "scalar", again, so that it can be easily changed. This is shown below.

```
      mc          Compressor maintenance cost (k$ per hp)          / 0.016 /
      pm          Pipeline maintenance facter (%TAC)               / 0.005 /
* Set mc and pm to 0 if maintenance is included in the operating cost
```

As mentioned before, it is not uncommon for compressor maintenance to be included in operating cost. If this is the case, the scalar value should be set to zero. This is directed by the comment. Next, this value is added into the economics. This is done so by adding the factor into the operating cost equation. From this perspective, it can be seen that a value of "0" negates the effect of compressor maintenance cost.

```
cmaintcost(t)            ..cmcost(t)=E=mc*(sum(s,W(s,t))+sum(c,WC(c,t)));
```

As can be seen, the compressor maintenance cost is simply the sum of the total work being performed by all of the compressors at any given time in hp multiplied by the compressor maintenance cost multiplier which is in k$/hp.

The pipeline and compressor maintenance costs are then combined into one maintenance term.

```
maintcost(t)             ..mcost(t)=E=pmcost(t)+cmcost(t);
```

This term can then be added to the other economic equations which involve cost.

```
NetPresentValeu          ..NPV=E=sum(t,((Rev(t)-Pipe(t)-Compr(t)-mcost(t)-icost(t)-Oper(t))/((1+i*eval)**(ord(t))))));
TotalAnnualCost(t)       ..TAC(t)=E=Pipe(t)+Compr(t)+mcost(t)+icost(t)+Oper(t);
```

Coating Cost

The coating cost values are implemented as a "parameter". This is shown below.

```
      CC(d) Cost of coating (k$ per Km)
*Basis: 6.5 $/ft. for 36" OD; assumes all pipe is coated
         /
              2     1.184748
              4     2.369495
              6     3.554243
              8     4.738991
              10    5.923739
              12    7.108486
              14    8.293234
              16    9.477982
              18    10.66273
              20    11.84748
              22    13.03223
              24    14.21697
              26    15.40172
              28    16.58647
              30    17.77122
              32    18.95596
              34    20.14071
              36    21.32546
              38    22.51021
              40    23.69495
              42    24.87970
                                  /
```

This new parameter must then be implemented into the economics of the program. This is shown below.

```
Pipecost(t)    ..Pipe(t)=E=sum((c,ca),LCC(c,ca)*sum(d,YDCC(c,ca,d,t)*(CP(d)+qr1+CC(d)+CT(d)+PIC(d))))
                                + sum((s,c),LSC(s,c)*sum(d,YDSC(s,c,d,t)*(CP(d)+qr1+CC(d)+CT(d)+PIC(d))));
```

As can be seen, the coating cost is simply added to the cost of the pipe by multiplying the coating cost in k$/km by the length in km of a given section of pipe.

Transportation Cost

The transportation cost values are implemented as a "parameter". This is shown below.

```
      CT(d) Cost of transportation (k$ per Km)
*Basis: 6 $/ft. for 36" OD
         /
              2     1.093613
              4     2.187227
              6     3.280840
              8     4.374453
              10    5.468067
              12    6.561680
              14    7.655293
              16    8.748906
              18    9.842520
              20    10.93613
              22    12.02975
              24    13.12336
              26    14.21697
              28    15.31059
              30    16.40420
              32    17.49781
              34    18.59143
              36    19.68504
              38    20.77865
              40    21.87227
              42    22.96588
                                  /
```

This new parameter must then be implemented into the economics of the program. This is shown below.

```
Pipecost(t)    ..Pipe(t)=E=sum((c,ca),LCC(c,ca)*sum(d,YDCC(c,ca,d,t)*(CP(d)+qrl+CC(d)+CT(d)+PIC(d))))
                        + sum((s,c),LSC(s,c)*sum(d,YDSC(s,c,d,t)*(CP(d)+qrl+CC(d)+CT(d)+PIC(d))));
```

As can be seen, the transportation cost is simply added to the cost of the pipe by multiplying the transportation cost in k$/km by the length in km of a given section of pipe.

Quadruple Random Length Joint Cost

Quadruple random length joint cost is implemented in much the same manner as pipeline maintenance cost. It is implemented as a "scalar", again, so that it can be easily changed. This is shown below.

```
      qrl        Cost of quadruple random length joints (k$ per km)          / 1.804461945 /
* Set qrl to 0 if quadrupal random length joints are not used
```

As shown, the option is present such that the method of adding quadruple random length joints can be toggled on and off by using a cost value or "0".

Next, this value is added into the economics. This is done so by adding the factor into the pipe cost equation. From this perspective, it can be seen that a value of "0" negates the effect of quadruple random length joint cost on the pipe cost.

```
Pipecost(t)    ..Pipe(t)=E=sum((c,ca),LCC(c,ca)*sum(d,YDCC(c,ca,d,t)*(CP(d)+qrl+CC(d)+CT(d)+PIC(d))))
                        + sum((s,c),LSC(s,c)*sum(d,YDSC(s,c,d,t)*(CP(d)+qrl+CC(d)+CT(d)+PIC(d))));
```

As can be seen, the compressor maintenance cost is simply added to the operating cost of the pipe by multiplying the compressor maintenance cost in k$/hp by the horsepower of a given compressor in hp.

Model Expansion Conclusion

Prior to this study, the mathematical linear pipeline optimization model lacked the ability to handle these parameters. The mathematical linear model for advanced pipeline design is now capable of handling more diverse pipeline diameters and contains many more economic options.

Appendix D

Linear Algebraic Model

Net Present Total Annual Cost

$$TAC(t) = Pipe(t) + Comp(t) + Maint(t) + Oper(t)$$

$$NPTAC = \sum_{t} \frac{TAC(t)}{(1 + i * eval)^{ord(t)}}$$

NPTAC = net present total annual cost
Pipe(t) = pipe cost
Comp(t) = compressor cost
Pmaint(t) = pipeline maintenance cost
Cmaint(t) = compressor maintenance cost
Icost(t) = instrumentation cost
Eval = number of years in one period
Ord(t) = gives the order of t (t1=1, t2=2, t3=3, …)
Card(y) = number of years

Net Present Value

$$NPV = \left(\sum_{t} \frac{Rev(t) - (pipe(t) + comp(t) + maint(t) - oper(t))}{(1 + i * eval)^{(ord(t))}} \right)$$

NPV = net present value
Rev(t) = revenue
Oper(t) = operating cost

Pipe Cost

$$pipe(t) = \sum_{s,c,ca} \left(length(s,c,ca) * \left(\sum_{d} YD(s,c,ca,d,t) * (steel(d) + coating(d) + trans(d) + install(d) + qrl) \right) \right)$$

S = supplier

C = consumer
Length (s,c,ca)= pipe length
D = diameter
YD(s,c,ca,d,t) = pipe installation at time t
Steel(d) = steel cost per length
Coating(d) = coating cost per length
Trans(d) = transportation cost per length
Install(d) = installation cost per length
Qrl = quadruple random length joints cost per length

Compressor Cost

$$comp(t) = cca * \left(\sum_{s} caps(s,t) + \sum_{c} capc(c,t) \right) + ccl * \left(\sum_{s} YCS(s,t) + \sum_{c} YCC(c,t) \right)$$

$$scompc = caps * cca + YCS * ccl$$
$$ccompc = capc * cca + YCC * ccl$$

Scompc(s,t) = cost of compressors at suppliers
Ccompc(c,t) = cost of compressors at consumers
Cca = angular compressor cost coefficient
Ccl = linear compressor cost coefficient
Caps(s,t) = capacity of a compressor at a supplier
Capc(c,t) = capacity of a compressor at a consumer
YCS(s,t) = supplier compressor installation binary
YCC(c,t) = consumer compressor installation binary

Pipeline Maintenance Cost

$alias(t,tt)$

$$maint(t) = mc * \left(\sum_{s} (Caps(s,t)) + \sum_{c} (Capc(c,t)) \right) + \left[pm * \sum_{tt=1 \to t} TAC(tt)\,(1 + i * eval)^{(ord(t) - ord(tt))} \right]$$

Pm = pipeline maintenance coefficient (%TAC)

Compressor Maintenance Cost

$$cmaint(t) = mc * \left(\sum_{s} (W(s,t)) + \sum_{c} (WC(c,t)) \right)$$

Mc = compressor maintenance coefficient ($/hp*hr)
W(s,t) = compressor work at a supplier
WC(c,t) = compressor work at a consumer

Revenue

$$Rev(t) = \left[eval * days * \left(\sum_{c} \left(QC(c,t) * Cprice(c)\right) - \sum_{s} \left(QSC(s,t) * Sprice(s)\right) \right) \right] - oper(t) - penal(t)$$

Days = # of operating days in a year
QC(c,t) = flowrate sold to consumer c
Cprice(c) = consumer price
QSC(s,t) = supplier flowrate
Sprice(s) = supplier price
Oper(t) = operating cost
Penal(t) = penalty cost

Compressor Capacity

$$0 * YCS(s,t) \leq caps(s,t) \leq MCaps(s,t) * YCS(s,t)$$
$$0 * YCC(c,t) \leq capc(c,t) \leq MCapc(c,t) * YCC(c,t)$$

Caps(s,t) = supplier capacity
Capc(c,t) = consumer capacity
Mcaps(s,t) = maximum supplier capacity
Mcapc(c,t) = maximum consumer capacity

Compressor Maximum Capacity

$$MCaps(s,t) = PW_{max}(s,p) * UB_{max}(t)$$
$$MCapc(c,t) = PWC_{max}(p,pa) * UB_{max}(t)$$

PW(s,p) = supplier compressor power
PWC(p,pa) = consumer compressor power
UB(t) = maximum flowrate at any time

Compressor Power

$$PW(s,p) = (3.02E-3) * k(p) * ST(s) * \frac{\left(\frac{P(p)}{SP(s)}\right)^{\left(\frac{Z(p)*(k(p)-1)}{k(p)}\right)} - 1}{(k(p)-1) * eff}$$

$$PWC(p,pa) = (3.02E-3) * k(p) * Tamb * \frac{\left(\frac{P_{i+1}(pa)}{P_i(p)}\right)^{\left(\frac{Z(p)*(k(p)-1)}{k(p)}\right)} - 1}{(k(p)-1) * eff}$$

K(p) = average natural gas compression ratio
ST(s) = supplier temperature
Tamb = ambient temperature
P(p) = pressure
SP(s) = supplier pressure
Z(p) = average compressibility factor
Eff = compressor efficiency

Maximum Flowrate based on Demand

$$UB(t) = \sum_{c} demt(c,t)$$

Demt(c,t) = consumer demand as a function of time

Consumer Demand

$$Summer1\text{: } demt_{t=0}(c,t) = 0.834877 * dem(c)$$
$$Winter1\text{: } demt_{t=0}(c,t) = 1.165123 * dem(c)$$
$$Summer\text{: } demt(c,t) = \left(1 + \left(0.01 * demrate(c)\right)\right) * demt_{t-2}(c)$$
$$Winter\text{: } demt(c,t) = \left(1 + \left(0.01 * demrate(c)\right)\right) * demt_{t-2}(c)$$

Dem(c) = initial consumer demand
Demrate(c) = annual consumer demand increase

Discrete Pressures

$$P(p) = Pmin + \frac{(ord(p) - 1) * (Pmax - Pmin)}{card(p) - 1}$$

Pmin = minimum operating pressure
Pmax = maximum operating pressure
Ord(p) = gives the order of p (p1=1, p2=1, p3=3,…)
Card(p) = number of discrete pressures

Compressor Work

$$W(s,t) - \left(PW(s,p) * \sum_{c} QSC(s,c,t) \right) \geq MCaps(s) * (XSP(s,t,p) - 1)$$

$$W(s,t) \leq \sum_{t} Caps(s,ta)$$

$$WC(c,t) - \left(PWC(c,p) * \sum_{c} QCC(c,ca,t) \right) \geq MCapc(c) * (XCC(c,t,p,pa) - 1)$$

$$WC(s,t) \leq \sum_{t} Capc\ (c,ta)$$

W(s,t) = work at a supplier compressor
WC(c,t) = work at a consumer compressor
QSC(s,c,t) = flowrate between a supplier and a consumer
QCC(c,ca,t) = flowrate between a consumer and a consumer
XSP(s,t,p) = binary existence of a supply pressure
XCC(c,t,p,pa) = binary existence of a pipe between two consumers

Energy Balance Restraints on Flowrate

$$\sum_d \left[\sum_p (XCC(c,t,p,pa) * FQCC(c,ca,d,p-1,pa) \right] \leq QCC(c,ca,t)$$
$$\leq \sum_d \left[\sum_p \left(XCC(c,t,p,pa) * FQCC_p(c,ca,d,p,pa) \right) \right]$$
$$\sum_d \left[\sum_p (XSC(s,t,p,pa) * FQSC(s,c,d,p-1,pa)) \right] \leq QSC(s,c,t)$$
$$\leq \sum_d \left[\sum_p \left(XSC(c,t,p,pa) * FQSC_p(s,c,d,p,pa) \right) \right]$$

XCC(c,t,p,pa) = binary existence of a pipe between two consumers
XSC(s,t,p,pa) = binary existence of a pipe between a consumer and a supplier
FQCC(c,ca,d,p,pa) = flowrate between two consumers based upon the energy balance
FQSC(s,c,d,p,pa) = flowrate between a consumer and supplier based upon the energy balance

Flowrates from Energy Balance

$$FQCC(c,ca,d,p,pa) = \frac{DPDZc(c,ca,d,p,pa) * ID(d)^5}{length(c,ca)^{0.5}}$$
$$FQSC(s,c,d,p,pa) = \frac{DPDZs(s,c,d,p,pa) * ID(d)^5}{length(s,c)^{0.5}}$$

DPDZc(c,ca,d,p,pa) = parameter with delta P and delta Z terms for the energy balance relating to consumers
DPDZs(s,c,d,p,pa) = parameter with delta P and delta Z terms for the energy balance relating to suppliers
ID = pipe inner diameter

Mathematical Model

$$DPDZc(c,ca,d,p,pa) = AC(d,c,ca) * \left(P_n^2(p) - P_{n+1}^2(p)\right) + BC(d,c,ca) * (ZC_{n+1}(ca) - ZC_n(c))$$

$$DPDZs(s,c,d,p,pa) = AS(d,s,c) * \left(P_n^2(p) - P_{n+1}^2(p)\right) + BS(d,s,c) * (ZC_{n+1}(c) - ZS_n(s))$$

AC(d,c,ca) = constant from simulated data
BC(d,c,ca) = constant from simulated data
AS(d,s,c) = constant from simulated data
BS(d,s,c) = constant from simulated data
ZC(c) = consumer elevation
ZS(s) = supplier elevation

Operating Cost

$$oper(t) = eval * oh * oc * \left(\sum_s W(s,t) + \sum_c WC(c,t)\right)$$

Oh = operating hours
Oc = operating cost as a function of compressor power and operating time

Penalty Cost

$$\begin{aligned} penal(t) = eval * days \\ * \Bigg[\left(\sum_s \left(amount(s,t) - \sum_c QSC\,(s,c,t)\right) * Spen(s)\right) \\ + \left(\sum_c (demt(c,ca,t) - QC(c,t)) * Cpen(c)\right)\Bigg] \end{aligned}$$

Days = # of operating days in a year
Amount = suggested agreed amount to buy
Spen (s) = supplier penalty for not delivering the agreed upon amount
Cpen(c) = consumer penalty for not buying the agreed upon amount

Flowrate Sold

$QC(c,t) = demt(c,ca,t)$
$QC(c,t) \leq demt(c,ca,t)$

The second equation can be used to justify not meeting the demand and except penalties if it is more economically preferred.

Suggested Amount to Buy

$$\left(amount(s,t) - \sum_{s} QSC\,(s,c,t)\right) \geq 0$$

Density

$den(p) = alpha * P(p)$

Den(p) = density
Alpha = angular coefficient of density equation
P(p) = discrete pressure

Maximum Flowrate based on Pipe Size

$FQVmax(d,p) = 86400 * EC * pi * ((0.0254 * ID(d))^2) * (den(p)^{0.5})$

FQVmax(d,p) = maximum flowrate from the maximum velocity
EC = erosion parameter
Pi = π
ID(d) = pipe inner diameter
Den(p) = density

Bound/Fixed Values

$QCC(c,c,t) = 0$

There is no flow from a consumer to itself.

$XDCC(c,c,d,t,p,pa) = 0$

There is no pipe from a consumer to itself.

$$XDCC(c,ca,d,t,p,pa)_{p<pa} = 0$$
$$XDSC(s,c,d,t,p,pa)_{p<pa} = 0$$
$$XCC(c,t,p,pa)_{p<pa} = 0$$

Compressors are not installed when p<pa.

Design Option Equations

$$\sum_{d} YDCC_{t=1}(c,ca,d,t) \leq XPCC(c,ca) \quad (for\ design\ options\ not\ 4)$$
$$\sum_{d} YDSC_{t=1}(s,c,d,t) \leq XPSC(s,c) \quad (for\ design\ options\ less\ than\ 4)$$

The number of pipes initially must be less than the desired number of pipes.

$$\sum_{d} YDCC(c,ca,d,t>1) = 0 \quad (for\ design\ options\ less\ than\ 3\ and\ after\ the\ first\ period)$$
$$\sum_{d} YDSC(s,c,d,t>1) = 0 \quad (for\ design\ options\ less\ than\ 3\ and\ after\ the\ first\ period)$$

No additions are made after the first period.

$$YCS_{t=1}(s,t) \leq 1 \quad (for\ design\ option\ 1)$$

If a compressor is to be installed, it must be installed within the first time period.

$$YCS(s,t)_{t>1} = 0 \quad (for\ design\ option\ 1\ and\ after\ the\ first\ period)$$
$$YCC(c,t)_{t>1} = 0 \quad (for\ design\ option\ 1\ and\ after\ the\ first\ period)$$

There are no compressors installed after the first time period.

Flowrate Balance

$$\sum_{s} QSC_{s\to c}(s,c,t) + \sum_{ca} QCC_{c\leftarrow ca}(c,c,t) = QC(c,t) + \sum_{ca} QCC_{c\to ca}(c,c,t)$$

QCS(s,c,t) = flow between supplier and consumer
QCC(c,c,t) = flow between consumers
QC(c,t) = flow sold

Operating Condition Relations

$$\sum_{p} XDCC(c, ca, d, t, p, pa) \leq \sum_{ta} YDCC(c, ca, d, t)$$
$$\sum_{p} XDSC(s, c, d, t, p, pa) \leq \sum_{ta} YDSC(s, c, d, t)$$

The number of pipes at all discrete pressures must be less than the actual number of pipes at any time.

Construction Timing

$$\sum_{d,t} YDCC(c, ca, d, t) \leq XPCC\,(c, ca)$$
$$\sum_{d,t} YDSC(s, c, d, t) \leq XPSC(s, c)$$

The number of pipes at a specific location at any time cannot be greater than the number of pipes at that location as indicated by the binary input of the model.

Demand Restraints on Flow Rate

$$QCC(c, ca, t) \leq UB(t) * \sum_{d}\left(\sum_{ta} YDCC\,(c, ca, d, ta \leq t)\right)$$
$$QSC(s, c, t) \leq UB(t) * \sum_{d}\left(\sum_{ta} YDSC\,(s, c, d, ta \leq t)\right)$$

The actual flowrate through any pipe and at any time cannot exceed the total demand flowrate at that time. Also, flow cannot exist if the pipe has not been installed yet.

Compressor Operating Condition Relations

$$\sum_{p} XCC(c, t, p, pa) \leq \sum_{ta} YCC(c, ta \leq t)$$

The number of compressors installed must be greater than or equal to the number of locations which require work. Also, work cannot exist if a compressor has not been installed yet.

Work Existence Relations

$$WC(c,t) \geq PWC(p,pa) * \sum_{ca} [QCC(c,ca,t) + (Mcapc * (PCin(c,t,p) + PCout(c,t,pa) + XYCC(c,t) - 3))]$$

$$XYCC(c,t) \leq \sum_{ta} YCC(c,ta)$$

PCin(c,t,p) = existence of a pressure into a consumer compressor
PCout(c,t,pa) = existence of a pressure out of a consumer compressor
XYCC(c,t) = not defined in the program

The work at a consumer must be greater than the power at any discrete pressure times the total flowrate of the system.

Compressor Installation Timing

$$\sum_{t} YCS(s,t) \leq PXS(s)$$

$$\sum_{t} YCC(c,t) \leq PXC(c)$$

The number of compressors at a location at any time must be less than or equal to the number of compressors allowed at that location by the user.

Supplier Pressure Relations

$$PSup(s,t,p) \leq \sum_{d,c} \left(\sum_{ta} YDSC(s,c,d,ta) \right)$$

$$PSup(s,t,p) \leq \sum_{d,c,pa} XDSC(s,c,d,t,p,pa)$$

The existence of an outlet pressure from a supplier compressor can only exist if there is a pipe connecting the supplier to a consumer.

$$PSup(s,t,p) \geq \sum_{d,pa} XDSC(s,c,d,t,p,pa)$$

The existence of an outlet pressure from a supplier compressor must exist if there is a pipe connecting the supplier to a consumer.

$$\sum_{p} PSup(s,t,p) \leq \sum_{d,c}\left(\sum_{ta} YDSC(s,c,d,ta)\right)$$

The total number of outlet pressures from all supplier compressors at each discrete pressure must be less than or equal to the total number of pipes connecting suppliers to compressors.

Compressor Inlet Pressure Relations

$$PCin(c,t,p) \geq \sum_{pa} XCC\,(c,t,p,pa)$$

If an inlet pressure to a consumer compressor exists, then there will be work required at that consumer.

$$PCin(c,t,p) \geq \sum_{d} XDSC(s,c,d,t,pa,p)$$

$$PCin(c,t,p) \geq \sum_{d} XDCC(ca,c,d,t,pa,p)$$

If an inlet pressure to a consumer compressor exists, then there must be a pipe connecting this compressor to some other location.

$$\sum_{p} PCin(c,t,p) = 1$$

There can only be one inlet pressure to a consumer compressor.

Compressor Outlet Pressure Relations

$$PCout(c,t,p) \geq \sum_{pa} XCC(c,t,pa,p)$$

If an outlet pressure to a consumer compressor exists, then there will be work required at that consumer.

$$PCout(c,t,p) \geq \sum_{d} XDCC\,(c,ca,d,t,p,pa)$$

If an outlet pressure to a consumer compressor exists, then there must be a pipe connecting this compressor to some other location.

$$\sum_{p} PCout(c,t,p) = 1$$

There can only be one outlet pressure from a consumer compressor.

Middle Compressor Pressure Relations

$$PCin(c,t,p) \leq PCout(c,t,p) + XYCC(c,t)$$
$$PCin(c,t,p) \geq PCout(c,t,p) - XYCC(c,t)$$
$$PCin(c,t,p) \geq \sum_{p} PCout(c,t,pa)$$

The number of outlet pressures must be equal to the number of inlet pressures at any consumer compressor.

Maximum Velocity Relations

$$4 * 1E3 * QCC(c,ca,t) * dens(p) \leq tolerance * \sum_{d,p} (XDCC(c,ca,d,t,p,pa) * FQVmax(d,p))$$
$$4 * 1E3 * QSC(s,c,t) * dens(p) \leq tolerance * \sum_{d,p} (XDSC(s,c,d,t,p,pa) * FQVmax(d,p))$$

The velocity through a pipe cannot exceed the maximum velocity based on the pipe parameters.

Agreed Amount to Purchase Relation

$$Amount(s,t) - \sum_{c} QCS(s,c,t) \geq 0$$

The supplier cannot deliver more gas than it agreed to.

Nonlinear Algebraic Model

Net Present Total Annual Cost

$$TAC(t) = Pipe(t) + Comp(t) + Maint(t) + Oper(t)$$

$$NPTAC = \sum_{t} \frac{TAC(t)}{(1 + i * eval)^{ord(t)}}$$

NPTAC = net present total annual cost
Pipe(t) = pipe cost
Comp(t) = compressor cost
Pmaint(t) = pipeline maintenance cost
Cmaint(t) = compressor maintenance cost
Icost(t) = instrumentation cost
Eval = number of years in one period
Ord(t) = gives the order of t (t1=1, t2=2, t3=3, …)
Card(y) = number of years

Net Present Value

$$NPV = \left(\sum_{t} \frac{Rev(t) - (pipe(t) + comp(t) + pmaint(t) + cmaint(t))}{(1 + i * eval)^{(ord(t))}} \right)$$

NPV = net present value
Rev(t) = revenue

Pipe Cost

$$pipe(t) = \sum_{s,c,ca} \left(length(s,c,ca) * \left(\sum_{d} XD(s,c,ca,d,t) * (steel(d) + coating(d) + trans(d) + install(d) + qrl) \right) \right)$$

S = supplier
C = consumer
Length(s,c,ca) = pipe length
D = diameter
XD(s,c,ca,d,t) = pipe existence binary
Steel(d) = steel cost per length
Coating(d) = coating cost per length
Trans(d) = transportation cost per length
Install(d) = installation cost per length
Qrl = quadruple random length joints cost per length

Compressor Cost

$$comp(t) = \sum_{s} scompc(s,t) + \sum_{c} ccompc(c,t)$$

$$= cca * \left(\sum_{s} caps(s,t) + \sum_{c} capc(c,t)\right) + ccl * \left(\sum_{s} YCS(s,t) + \sum_{c} YCC(c,t)\right)$$

$$scompc(s,t) = scap(s,t) * cca + YCS(s,t) * ccl$$

$$ccompc(c,t) = ccap(c,t) * cca + YCC(c,t) * ccl$$

Scompc(s,t) = cost of compressors at suppliers
Ccompc(c,t) = cost of compressors at consumers
Cca = angular compressor cost coefficient
Ccl = linear compressor cost coefficient
Caps(s,t) = capacity of a compressor at a supplier
Capc(c,t) = capacity of a compressor at a consumer
YCS(s,t) = supplier compressor existence binary
YCC(c,t) = consumer compressor existence binary

Pipeline Maintenance Cost

$$alias(t,tt)$$

$$pmaint(t) = pm * \sum_{tt=1 \to t} TAC(tt)\,(1 + i * eval)^{(ord(t) - ord(tt))}$$

Pm = pipeline maintenance coefficient (%TAC)

Compressor Maintenance Cost

$$cmaint(t) = mc * \left(\sum_{s} (W(s,t)) + \sum_{c} (WC(c,t))\right)$$

Mc = compressor maintenance coefficient ($/hp*hr)
W(s,t) = compressor work at a supplier
WC(c,t) = compressor work at a consumer

Revenue

$$Rev(t) = \left[eval * days * \left(\sum_c \left(QC(c,t) * Cprice(c)\right) - \sum_c \left(QSC(s,c,t) * Sprice(s)\right)\right)\right] - oper(t) - penal(t)$$

Days = # of operating days in a year
QC(c,t) = flowrate sold to consumer c
Cprice(c) = consumer price
QSC(s,c,t) = supplier flowrate
Sprice(s) = supplier price
Oper(t) = operating cost
Penal(t) = penalty cost

Compressor Capacity

$$0 * XCS(s,t) \leq caps(s,t) \leq 1E8 * XCS(s,t)$$
$$0 * XCC(c,t) \leq capc(c,t) \leq 1E8 * XCC(c,t)$$

Caps(s,t) = supplier capacity
Capc(c,t) = consumer capacity
XCS(s,t) = compressor existence binary
XCC(c,t) = compressor existence binary

Maximum Flowrate based on Demand

$$UB(t) = \sum_c demt\,(c,t)$$

Demt(c,t) = consumer demand as a function of time

Consumer Demand

$$Summer1{:}\ demt_{t=0}(c,t) = 0.834877 * dem(c)$$
$$Winter1{:}\ demt_{t=0}(c,t) = 1.165123 * dem(c)$$
$$Summer{:}\ demt(c,t) = \left(1 + \left(0.01 * demrate(c)\right)\right) * demt_{t-2}(c)$$
$$Winter{:}\ demt(c,t) = \left(1 + \left(0.01 * demrate(c)\right)\right) * demt_{t-2}(c)$$

Dem(c) = initial consumer demand
Demrate(c) = annual consumer demand increase

Bound/Fixed Values

$$XDCC_{t>1}(c, ca, d, t) = 0$$
$$XDSC_{t>1}(s, c, d, t) = 0$$

No pipes are installed after the initial time period.

$$QCC_{max}(c, ca, t) = UB(t) * PXCC(c, ca)$$
$$QSC_{max}(s, c, t) = UB(t) * PXSC(s, c)$$

The maximum flow through a pipe is the maximum demand flowrate.

$$SP(s) \leq DP(s, t) \leq P_{max}$$

The pressure drop through any pipe must be between the supply pressure and the maximum operating pressure.

$$P_{min} \leq Pin(c, t) \leq P_{max}$$
$$P_{min} \leq Pout(c, t) \leq P_{max}$$

The pressure into and out of a compressor must fall between the minimum and maximum operating pressures.

Flowrate Balance

$$\sum_{s} QSC_{s\to c}(s, c, t) + \sum_{ca} QCC_{c\leftarrow ca}(c, c, t) - \sum_{ca} QCC_{c\to ca}(c, c, t) = QC(c, t)$$

QCS(s,c,t) = flow between supplier and consumer
QCC(c,c,t) = flow between consumers
QC(c,t) = flow sold

Flowrate Sold

$$QC(c, t) = demt(c, ca, t)$$
$$QC(c, t) \leq demt(c, ca, t)$$

The second equation can be used to justify not meeting the demand and except penalties if it is more economically preferred.

Mathematical Model

$$\sum_{d}\left(\sum_{ta}(((AC(d,c,ca)*(Pout^2(c,t)-Pin^2(ca,t))+BC(d,c,ca)*\left(ZC_{ca}(ca)-ZC_c(c)\right))*ID(d)^5 * XDCC(c,ca,d,ta))\right)=QCC(c,ca,t)*length(c,ca)$$

$$\sum_{d}\left(\sum_{ta}(((AS(d,s,c)*(DP^2(s,t)-Pin^2(c,t))+BS(d,s,c)*\left(ZC(s)-ZS(c)\right))*ID(d)^5 * XDSC(s,c,d,ta))\right)=QSC(s,c,t)*length(s,c)$$

AC(d,c,ca) = constant from simulated data
BC(d,c,ca) = constant from simulated data
AS(d,s,c) = constant from simulated data
BS(d,s,c) = constant from simulated data
ZC(c) = consumer elevation
ZS(s) = supplier elevation
XDCC(c,ca,d,ta) = Consumer-to-consumer pipe existence binary
XDSC(s,c,d,ta) = Supplier-to-consumer pipe existence binary

Construction Timing

$$\sum_{d} XDCC(c,ca,d,t) \leq PXCC(c,ca)$$

$$\sum_{d} XDSC(s,c,d,t) \leq PXSC(s,c)$$

The number of pipes at a specific location at any time cannot be greater than the number of pipes at that location as indicated by the binary input of the model.

Demand Restraints on Flow Rate

$$QCC(c,ca,t) \leq UB(t) * \sum_{d}\left(\sum_{ta} XDCC(c,ca,d,ta \leq t)\right)$$

$$QSC(s,c,t) \leq UB(t) * \sum_{d}\left(\sum_{ta} XDSC(s,c,d,ta \leq t)\right)$$

The actual flowrate through any pipe and at any time cannot exceed the total demand flowrate at that time. Also, flow cannot exist if the pipe has not been installed yet.

Compressor Work

$$W(s,t) * (k-1) * eff = (3.02E-3) * \sum_{c} (QSC(s,c,t)) * k * ST(s) * \left(\left(\frac{DP(s,t)}{SP(s)} \right)^{\frac{Z*(k-1)}{k}} - 1 \right)$$

$$WC(c,t) * (k-1) * eff = (3.02E-3) * \sum_{ca} (QCC(c,ca,t)) * k * Tamb * \left(\left(\frac{Pout(c,t)}{Pin(c,t)} \right)^{\frac{Z*(k-1)}{k}} - 1 \right)$$

k = average natural gas compression ratio
ST(s) = supplier temperature
Tamb = ambient temperature
DP(s,t) = pressure drop
SP(s) = supplier pressure
Pout(c,t) = compressor outlet pressure
Pin(c,t) = compressor inlet pressure
Z = average compressibility factor
Eff = compressor efficiency

Compressor Work

$$W(s,t) \leq \sum_{t} Caps(s,ta)$$

$$WC(c,t) \leq \sum_{t} Capc\,(c,ta)$$

Caps(s,ta) = supplier capacity
Capc(c,ta) = consumer capacity

Compressor Installation Timing

$$\sum_{t} XCS(s,t) \leq PXS(s)$$

$$\sum_{t} XCC(c,t) \leq PXC(c)$$

The number of compressors at a location at any time must be less than or equal to the number of compressors allowed at that location by the user.

Pressure Relations

$$SP(s) \leq DP(s,t) \leq P_{max} * \sum_{ta} \left(XCS(s,ta)\right) + SP(s)$$

The pressure drop at the supplier compressor must fall between the supply pressure and the maximum operating pressure plus the supply pressure.

$$Pin(c,t) \leq Pout(c,t) \leq P_{max} * \sum_{ta} (XCC(c,ta))$$

The outlet pressure of a consumer compressor must fall between the inlet pressure of that compressor and the maximum operating pressure.

Maximum Velocity Relations

$$\begin{aligned} &4 * 1E3 * QCC(c,ca,t) * dens \\ &\qquad \leq 3.66 * 86400 * pi * \sum_{d} \left(\sum_{ta} \left(\left(0.0254 * ID(d)\right)^2 \right) XDCC(c,ca,d,ta) \right) * alpha \\ &\qquad * Pout(c,t) \end{aligned}$$

$$\begin{aligned} &4 * 1E3 * QSC(s,c,t) * dens \\ &\qquad \leq 3.66 * 86400 * pi * \sum_{d} \left(\sum_{ta} \left(\left(0.0254 * ID(d)\right)^2 \right) XDSC(s,c,d,ta) \right) * alpha \\ &\qquad * DP(s,t) \end{aligned}$$

The velocity through a pipe cannot exceed the maximum velocity based on the pipe parameters.

Agreed Amount to Purchase Relation

$$Amount(s,t) - \sum_{c} QCS(s,c,t) \geq 0$$

The supplier cannot deliver more gas than it agreed to.

Appendix E

A How-To Guide for Inputting a Model into the Program

Sets

Sets are the first things seen in the mathematical programming model. An image of these sets is shown below. All images show the input for the case study mentioned in this paper.

Linear

```
Set s  supliers     / 1*1 /
    c  consumers    / 1*5 /
    y  life time of the project   /1*8/
    t  evaluations in the life time of the project   /1*16/
    p  pressures /1*9/
    d  pipe diameter options /2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42/;
```

Nonlinear

```
Set s  supliers     / 1*1 /
    c  consumers    / 1*5 /
    y  life time of the project   /1*8/
    t  evaluations in the life time of the project   /1*16/
    r  random numbers /1*1000/
    d  pipe diameter options /2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42/;
```

It is seen that there is one supplier and five consumers. The project is 8 years long and has an evaluation every 6 months. The linear model evaluates at 9 discrete pressures, and the nonlinear model includes a random number generator which is used in some solving options. Next are the diameters. All diameter options which are to be considered are defined here.

Aliases

Next, the aliases are assigned. An image is provided below.

```
Alias (c,ca),(t,ta,tt),(p,pa,pp);
```

An alias allows multiple variables to mean essentially the same thing. In this case, compressors can be called ‘c’ or ‘ca’, times can be defined by ‘t’, ‘ta’, or tt’, and pressures can be called ‘p’, ‘pa’, or ‘pp’.

Scalars

Next, the scalar inputs are added as shown below.

```
Scalar
      ccl       Linear coefficient of compressors cost equation (k$)                / 2370 /
      cca       Angular coefficient of compressors cost equation (k$ per hp)       / 1.42173 /
      dens      Density on the natural gas at standart conditions (kg per m3)      / 0.797 /
      EC        Erosion parameter (for v[=]m per s - dens[=]kg per m3)             / 123 /
      tolerance Allowed percentage of erosion velocity                              / 1 /
      oc        Operating cost (k$ per hp*hr)                                       / 0.000031 /
*      qrl       Cost of quadruple random length joints (k$ per km)                 / 0 /
      qrl       Cost of quadruple random length joints (k$ per km)                  / 1.804461945 /
* Set qrl to 0 if quadrupal random length joints are not used
*      mc        Compressor maintenance cost (k$ per hp)                            / 0 /
*      pm        Pipeline maintenance facter ($TAC)                                 / 0 /
      mc        Compressor maintenance cost (k$ per hp)                             / 0.016 /
      pm        Pipeline maintenance facter (%TAC)                                  / 0.005 /
* Set mc and pm to 0 if maintenance is included in the operating cost
      oh        Operating hours in a year                                           / 8640 /
      days      Operating days in a year                                            / 360 /
      eff       Compressor efficiency                                               / 0.8 /
      i         Interest rate                                                       / 0.08 /
      Ps        Standad pressure (kPa)                                              / 101.325 /
      Ts        Standar temperature (R)                                             / 491.67 /
      Tamb      Ambient temperature (R)                                             / 524.07 /
      alpha     Angular coefficient of density equation                             / 0.0079 /
      pi        Pi                                                                  / 3.14 /
      Zz        Avarage compresibility factor of the natural gas                    / 0.941 /
      kk        Avarege compression ratio of the natural gas                        / 1.4 /
      cpgas     Specific heat capacity (KJ per Kg K)                                / 2.48 /
      Factor       Factor to account other costs                                    / 0.2 /
*Add here the maximum and minimum pressure that your system is allowed to have
*750psi=5171.07kPa
     MaxPressure  Maximum operation pressure(kPa)                                   /10050.5/
     MinPressure  Minimum operation pressure(kPa)                                   /4200/
```

There are many values here, but most of them have sufficient descriptions in blue. The ones that are not so clear are elaborated upon here. ‘ccl’ and ‘cca’ are coefficients used to price the compressors. A linear relationship is assumed between compressor capacity and compressor cost. The slope of this relationship is ‘cca’, and the y-intercept is ‘ccl’. The ‘Factor’ was not used in our case study, but it can be used if little knowledge is known about some of the finer details of the economic options. One unique aspect of this data is that it is rather simple to switch gases. The user would simply need to update ‘dens’, ‘Zz’, ‘Kk’, and ‘cpgas’. The program should run just fine with this new gas so long as it is in fact a single phase gas.

Parameters

Next, the actual identity of the network is implemented as shown below.

```
Parameters

      SP(s)  Suppliers pressure (kPa)
           / 1   3548.7 /
      ST(s)  Suppliers temperature (R)
           / 1   529.67 /
      Sprice(s) Suppliers price (k$ per 1E3 m^3)
           / 1  0 /
      Spenalty(s) Suppliers price ($ per m^3)
           / 1  0 /
      Dem(c) Demand of consumers (1E3 * m^3 per day)
           / 1    283.17
             2    566.34
             3    0
             4    2831.7
             5    1699.0 /

     Demrate(c) Rate in which demand increases annualy (%)
           / 1   10
             2   10
             3   10
             4   10
             5   10 /

     Cprice(c) Consumers price (k$ per 1E3 m^3)
           / 1   0.32
             2   0.33
             3   0.00
             4   0.30
             5   0.30 /


     Cpenalty(c) Penalty applied when demand is not deliveried
           / 1   0
             2   0
             3   0
             4   0
             5   0  /

     PXS(s) Compressors are allowed at the supplier station
           / 1   1 /

     PXC(c) Compressors are allowed at the consumers station
           / 1   1
             2   1
             3   1
             4   1
             5   1 /
```

```
ZS(s) Elevation  (km)
      / 1   .185928 /

ZC(c) Elevetion (km)
      / 1   .36575
        2   .54864
        3   .2286
        4   .10668
        5   .12816 /
```

```
Table XPCC(c,c) Connections between compressor and pipe
        1  2  3  4  5
     1  0  1  0  0  0
     2  0  0  1  0  0
     3  0  0  0  1  1
     4  0  0  0  0  0
     5  0  0  0  0  0;

Table XPSC(s,c) Connections between compressor and pipe
        1  2  3  4  5
     1  1  0  0  0  0;

Table LSC(s,c) Distances between supplier and consumers (km)
         1      2      3      4      5
     1   80.47  1E5    1E5    1E5    1E5;

Table LCC(c,c) Distances between consumer and consumer (km)
          1      2      3      4      5
     1    1E5    96.56  1E5    1E5    1E5
     2    1E5    1E5    144.8  1E5    1E5
     3    1E5    1E5    1E5    128.7  32.19
     4    1E5    1E5    1E5    1E5    1E5
     5    1E5    1E5    1E5    1E5    1E5 ;
```

'Spenalty' and 'Cpenalty' are all zero. Our case study did not consider penalties for not meeting demand. A simple edit of the program allows it to choose to not meet demand and take a penalty if it is more economically sound than meeting demand. Table XPCC defines all connections from consumer to consumer in binary. Table XPSC defines all connections from supplier to consumer in binary. Table LSC expresses the distances between supplier and consumer. Table LCC expresses the distances between consumer and consumer. 1E5 is used for nonexistent distances. The program will read this large distance as infeasible.

Also included in this section are all of the pipe pricing specifications which are a function of diameter and are shown in an earlier appendix.

Positive/Binary Variables

These input simply tell the program which variables should be always positive or always binary. These should remain the same regardless of the system being applied to the model.

```
Positive variable W,WC,QCC,QSC,QC,amount,Comprs,Comprc,Pipe,Caps,Capc,Penal;
Binary variables YDCC,YDSC,XDCC,XDSC,XCS,XCC,YCS,YCC,PSup,PCin,Pcout,XYCC;
```

Demand

The demand equations can be quite tricky. They must be updated depending on both the demand variations as well as the lifetime of the project. The equations shown express the seasonal variation, the 10% annual demand increase, and the 8-year project lifetime.

```
Demt(c,'1')=0.834877*Dem(c);
Demt(c,'2')=1.165123*Dem(c);
*$ontext
Demt(c,'3')=(1+(0.01*demrate(c)))*Demt(c,'1');
Demt(c,'4')=(1+(0.01*demrate(c)))*Demt(c,'2');
*$ontext
Demt(c,'5')=(1+(0.01*demrate(c)))*Demt(c,'3');
Demt(c,'6')=(1+(0.01*demrate(c)))*Demt(c,'4');
*$ontext
Demt(c,'7')=(1+(0.01*demrate(c)))*Demt(c,'5');
Demt(c,'8')=(1+(0.01*demrate(c)))*Demt(c,'6');
Demt(c,'9')=(1+(0.01*demrate(c)))*Demt(c,'7');
Demt(c,'10')=(1+(0.01*demrate(c)))*Demt(c,'8');
*$ontext
Demt(c,'11')=(1+(0.01*demrate(c)))*Demt(c,'9');
Demt(c,'12')=(1+(0.01*demrate(c)))*Demt(c,'10');
Demt(c,'13')=(1+(0.01*demrate(c)))*Demt(c,'11');
Demt(c,'14')=(1+(0.01*demrate(c)))*Demt(c,'12');
Demt(c,'15')=(1+(0.01*demrate(c)))*Demt(c,'13');
Demt(c,'16')=(1+(0.01*demrate(c)))*Demt(c,'14');
```

After completing these steps, the program should be ready to run any model.

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